DTIC FILE COPY

SECURITY CLASSIFICATION OF THIS PAGE

(1)

REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS NONE
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED.
AD-A218 027	5. MONITORING ORGANIZATION REPORT NUMBER(S) AFIT/CI/CIA-89-124
6a. NAME OF PERFORMING ORGANIZATION AFIT STUDENT AT Univ of Houston	6b. OFFICE SYMBOL (If applicable)
7a. NAME OF MONITORING ORGANIZATION AFIT/CIA	
6c. ADDRESS (City, State, and ZIP Code)	7b. ADDRESS (City, State, and ZIP Code) Wright-Patterson AFB OH 45433-6583
8a. NAME OF FUNDING/SPONSORING ORGANIZATION	8b. OFFICE SYMBOL (If applicable)
9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER	
8c. ADDRESS (City, State, and ZIP Code)	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. / PROJECT NO. / TASK NO. / WORK UNIT ACCESSION NO.

11. TITLE (Include Security Classification) (UNCLASSIFIED)
DESIGN DECISION MAKING WITH MULTIPLE INTERACTING CRITERIA AND VARYING RELATIVE WEIGHTS

12. PERSONAL AUTHOR(S)
JACOB SIMONS

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
~~THESIS~~/DISSERTATION	FROM ______ TO ______	1989	347

16. SUPPLEMENTARY NOTATION
APPROVED FOR PUBLIC RELEASE IAW AFR 190-1
ERNEST A. HAYGOOD, 1st Lt, USAF
Executive Officer, Civilian Institution Programs

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

DTIC ELECTE FEB 13 1990 S D

D

BEST AVAILABLE COPY

90 02 12 019

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL ERNEST A. HAYGOOD, 1st Lt, USAF	22b. TELEPHONE (Include Area Code) (513) 255-2259 / 22c. OFFICE SYMBOL AFIT/CI

DD Form 1473, JUN 86 Previous editions are obsolete. SECURITY CLASSIFICATION OF THIS PAGE

AFIT/CI "OVERPRINT"

FROM: Major Jacob V. Simons, Jr.
10810 Telephone Rd., #342
Houston, TX 77075

4 Aug 89

SUBJECT: Dissertation Information

TO: AFIT/CISP

1. This letter responds to AFITR 53-1, paragraphs 7-7 and 7-8, concerning required dissertation information. The two abstracts required by para 7-7 and the unbound copy of the dissertation required by para 7-8 are enclosed.

2. Paragraph 7-7 also requests the following information:

a. Author: Jacob Van Vranken Simons, Jr.

b. Title: DESIGN DECISION MAKING WITH MULTIPLE INTERACTING CRITERIA AND VARYING RELATIVE WEIGHTS

c. Military rank and service branch: Major, USAF

d. Date (year only): 1989

e. Number of pages: 347 (including preliminary pages)

f. Degree awarded: Ph.D. (Business Administration)

g. Name of institution: University of Houston

3. Please note that the abstracts enclosed are slightly different from the abstract contained in the dissertation itself. These differences have been minimized and are due to the differences between the requirements of AFITR 53-1 and those of my university.

Jacob V Simons Jr.

JACOB V. SIMONS, JR., Maj, USAF

DESIGN DECISION MAKING WITH MULTIPLE INTERACTING CRITERIA AND VARYING RELATIVE WEIGHTS

A Dissertation

Presented to

the Faculty of the College of Business Administration

University of Houston

In Partial Fulfillment

Of the Requirements for the Degree

Doctor of Philosophy

by

Jacob Van Vranken Simons, Jr.

July, 1989

DESIGN DECISION MAKING WITH MULTIPLE INTERACTING CRITERIA

AND VARYING RELATIVE WEIGHTS

APPROVED:

Benjamin Ostrofsky

Benjamin Ostrofsky, Professor of
Decision and Information Sciences
Chairman of Committee

B. M. Khumawala

Basheer M. Khumawala, Professor of
Decision and Information Sciences

Everette S. Gardner, Jr.

Everette S. Gardner, Jr., Professor
of Decision and Information Sciences

Richard W. Scamell

Richard W. Scamell, Associate
Professor of Decision and Information
Sciences

John M. Ivancevich

John M. Ivancevich, Dean
College of Business Administration

To Carol, Trey, and Charlie,

the joy of my life.

I love you.

DESIGN DECISION MAKING WITH MULTIPLE INTERACTING CRITERIA

AND VARYING RELATIVE WEIGHTS

Abstract of a Dissertation

Presented to

the Faculty of the College of Business Administration

University of Houston

In Partial Fulfillment

Of the Requirements for the Degree

Doctor of Philosophy

by

Jacob Van Vranken Simons, Jr.

July, 1989

ACKNOWLEDGEMENTS

I respectfully acknowledge the contributions of the many people and organizations who helped make this research possible. To each I offer my deepest thanks. In particular, I offer my appreciation to:

My chairman, Dr. Benjamin Ostrofsky, and my committee members, Dr. Basheer M. Khumawala, Dr. Everette S. Gardner, Jr., and Dr. Richard W. Scamell for their invaluable guidance and unfailing support. I cannot conceive of a finer committee.

Doc Kiessling, for his wisdom and friendship.

Numerous faculty and colleagues at the University of Houston, for their advice and encouragement.

The U.S. Air Force in general and, specifically, the Air Force Institute of Technology, for giving me this opportunity and financial support.

NASA's Johnson Space Center, Mr. Walter Scott and Mr. Dave Young in particular, for their time and the demonstration in Chapter 5.

Billie and Jack Turner and Doris and Elmer Smith, who shared their homes to give me a productive work environment.

My parents, Jacob V. Simons, Sr. and Janet K. Simons, who gave me the tools and the motivation to succeed.

God almighty, without whom nothing is possible.

ABSTRACT

The design environment is typically characterized by conflicting objectives and numerous design alternatives, making selection of a specific system design problematic. The broad goal of this research is the development of a prescriptive decision making approach which facilitates the comparison of a large number of design alternatives with respect to multiple criteria. The approach developed considers both objective and subjective factors, yet strives for optimization rather than satisficing of the decision criteria. Each criterion is considered in light of perceived relative weights which are permitted to vary in response to the performance achieved for that objective. Finally, the decision technique described provides a means of treating potential interactions among the multiple criteria.

A design methodology framework is presented, containing state-of-the-art features stemming from the dramatic influence of systems theory. These characteristics include a broad, macro view of the system being designed, an expansionist, deductive perspective in the evaluation of design alternatives, and explicit consideration of system life cycle phases. Within the context of this framework methodology, major decision making approaches frequently suggested as alternatives for the design decision problem are reviewed and critiqued with respect to the desired features. Theses. (AW)

The criterion function approach is selected as the most promising starting point; however, the achievement of the research goal requires

the resolution of inconsistencies which occur using current implementation methods. The inconsistencies are specified, their sources identified, and their solutions proposed. A demonstration of the proposed methods is accomplished using the design of an auxiliary power unit system for NASA's space shuttle.

The primary contributions of this research are the improved computational methods in implementing the criterion function and the concurrent development and demonstration of a design decision making approach which achieves the stated desirable characteristics. Potential areas for future research include empirical studies concerning the use and success of design methodologies in practice, promotion of the criterion function's ability to consider intersection relative weights, exploration of the method for normalizing relative weights, and implementation of the criterion function in NASA design decision making.

TABLE OF CONTENTS

Appendix

LIST OF TABLES

Table

LIST OF FIGURES

Figure

Chapter 1

COMPLEX SYSTEM DESIGN

System Design/Planning

Throughout history, man's success has depended on his ability to master his environment. In some cases, physical strength and agility have been adequate to achieve this mastery. However, we generally attribute the majority of man's success to his unique mental capabilities. In particular, we consider man superior to his animal peers primarily in terms of his ability to understand and reason. Even in basic matters of survival, man frequently depends on his ability to analyze the issues relevant to a given situation and then synthesize a plan of action which will achieve some desired objective(s).

In the most elementary cases, activities may be planned and carried out in a simple, informal manner. However, as history has progressed, man's objectives have become increasingly sophisticated and more complex. Large undertakings such as governmental organization and technological innovation have required that many people, tools, and resources be integrated to achieve a unified goal.

Consequently, the effort and complexity of the planning activities have increased in a corresponding manner. For example, the building of the pyramids and the development of the Roman empire required far more intricate planning activities than anything which had gone before. Similarly, modern activities such as space programs and social welfare systems tax man's ability to reason far beyond what was previously possible. Therefore, as time has passed, the need for more effective, formal approaches to design and planning has continued to increase and can be expected to do so for the foreseeable future.

Perhaps more than ever, effective system design is perceived as key to the resolution of today's goals and problems:

> Systems methodology and design (SMD) is emerging as a concept of and reasoning principles for integrating traditional engineering problem solving with systems theory, management science, behavioral decision theory, and planning and design approaches. The resultant combination is a methodological and creative concept for attacking and solving, with a long range perspective, today's problems in organizations in general and in engineering and industry in particular. At a time when poor growth in productivity and management performance are being shown to be important obstacles to healthy economic growth, SMD might be an important concept for both short- and long-term recovery. (Nadler, 1985)

This research is another step in the on-going effort to provide the planning methods and tools necessary to perpetuate the achievement of man's goals. Since it is unreasonable to assume that all of mankind's goals have been explicitly defined, any methods developed as part of this research must be conceived and described in the broadest

manner possible in order to achieve flexibility of application. However, the potential ambiguity of discussing broad concepts suggests that we precede further discussion with the following definitions.

Definition of a System

Throughout this research, we will refer to the design or planning of systems without necessarily defining what those systems may be. The definition we choose to use for the term "system" is:

> Any set of objects and/or attributes and the relationships among them. (Ostrofsky, 1988)

This definition is intentionally obtuse. A system may be a physical object or collection of objects. By contrast, it may be no more than a set of operating procedures or rules. A system may include people, computer programs, or any combination of these possibilities. In fact, most large scale systems will normally include some combination of all.

We closely follow the orientation of systems theory in which it is generally assumed that a system exists for some purpose(s). Following this line of reasoning, a system is typically considered to involve some combination of inputs, outputs, and a transformation process which converts inputs to outputs. Although many representations are possible, we will generally refer to the systems model used by Asimow (1962) in which a system is described in terms of

its environmental inputs, intended inputs, desired outputs, and undesired outputs (Figure 1).

Definition of Design

In order to achieve a particular system, a series of purposeful activities must be defined and undertaken. We refer to this set of activities as system design. While the term "design" typically generates a mental image of some physical object or system, it must be remembered that our definition of a system is not so restrictive. Consequently, we consider the more generic term "planning" to be synonymous with design. The two terms will be used interchangeably in this research.

Significant Developments in Design

As the art and science of design have progressed through history, many developments have emerged which have left a lasting influence on current approaches (Asimow, 1962; Buhl, 1961; de Neufville and Marks, 1974; Department of Defense, 1968; Dixon, 1966; Gosling, 1962; Gregory, 1966; Hall, 1962, 1969; Haupt, 1978; Hill, 1970; Holzman, 1979; Jones, 1970; Kline; Meredith, 1973; Middendorf, 1969; Ostrofsky, 1977d; Sage, 1977, 1986; Simon, 1975; U.S. Air Force, 1966, 1974; Vidosic, 1969; Wallace, 1978; Wilson, 1965; Woodson, 1966; Wymore, 1976). Although these developments are too numerous to list exhaustively or describe individually, the following discussion

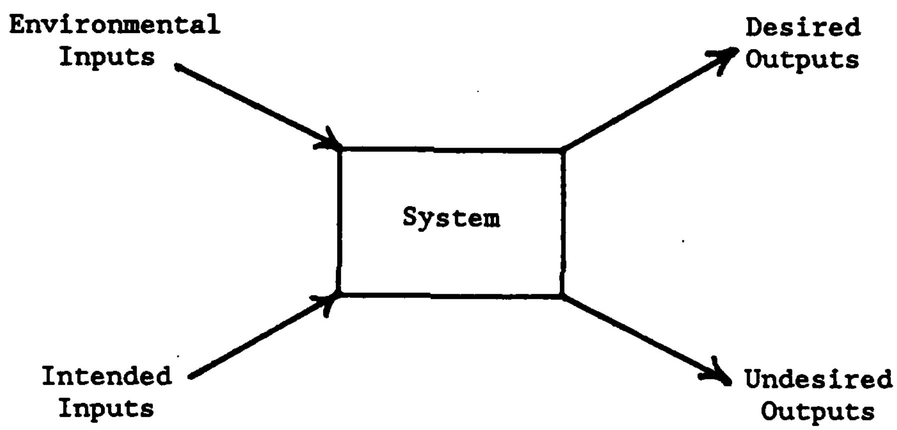


Figure 1. System Representation

highlights some of the significant issues emerging from the intense research devoted to this area in recent decades.

Design Phases

Given our description of design as a series of purposeful activities, it is logical that many researchers and authors have focused on defining the generic logical sequence and grouping of required activities. These groupings are typically referred to as design steps or phases.

Some of the more prominent delineations of design phases are shown in Figure 2. Two of the more influential schemes were those of Hall (1962) and Asimow (1962). As is the case with most such classification schemes, the earliest design stages are concerned with the broadest issues of the effort while the later stages serve to specify increasingly finer levels of detail. For example, stages such as program planning and feasibility study generally begin with a broad, yet formal statement of the problem and description of the relevant influences and participants in the design environment. Subsequent stages strive first for determination of a general solution approach and then for a detailed description of how that approach is to be implemented.

Similar in concept to those proposed by Hall and Asimow, the

Hall (1962)
Systems Engineering Phases

- Program Planning
- Project Planning
- System Development
- Studies During Development
- Current Engineering

Asimow (1962)
Design Phases

- Feasibility Study
- Preliminary Design
- Detail Design
- Planning for Production
- Planning for Distribution
- Planning for Consumption/ Operations
- Planning for Retirement

DoD (1968)
Design Phases

- Formulation
- Contract Definition
- Development
- Production
- Operational

Hall (1969)
Systems Engineering Phases

- Program Planning
- Project Planning
- System Development
- Production
- Distribution
- Operation
- Retirement

Wymore (1976)
Design Steps

- Problem Definition
- Preliminary Design
- Final Design
- Implementation
- Acceptance
- Testing
- Operation
- Retirement

Sage (1977)
Design Phases

- Program Planning
- Project Planning
- System Development
- Production
- Distribution
- Operation
- Retirement

Ostrofsky (1977d)
Design Phases

- Feasibility Study
- Preliminary Design
- Detail Design
- Production
- Distribution
- Consumption/Operations
- Retirement

Figure 2. Design Phases

(Adapted from Folkeson, 1982)

design phases specified by the Department of Defense (1968) were also chosen to highlight logical review points between phases, facilitating the monitoring of contracted design efforts common to Air Force system development.

Subsequent design phase descriptions (Woodson, 1966; Hall, 1969; Wymore, 1976; Sage, 1977; Ostrofsky, 1977d) were largely refinements of these earlier schemes. Although the phase names and the grouping of individual tasks and activities vary slightly from scheme to scheme, the overall content is quite similar. The general conclusion to be drawn from comparing the alternative descriptions is that the exact sequence and method of implementation is less important than the need to insure that the commonly agreed on tasks are accomplished (Wallace, 1988).

System Life Cycle Phases

Early attempts to define the design phases generally focused on visualizing the target system in steady state operation. The primary goal was perceived as maximizing the potential performance of the system during the operational portion of its existence. Gradually, experience led many designers to begin to recognize the importance of considering other phases of the system's life when first specifying what characteristics the system should possess. In particular, it became apparent that other characteristics of importance to the ultimate success of the system had to be considered from the outset of

the design effort and built in. Such characteristics might include the system's producability, ease of implementation/installation, reliability, recyclability, disposability, etc. Since these characteristics typically have a substantial impact on the requirement for and use of equipment, people, or other system resources they cannot be effectively or efficiently added on after-the-fact to an existing system.

As a result of these realizations, researchers sought to describe general phases in the life cycle of any intended system so that these phases could become the targets of specific planning activities during the design process (Asimow, 1962; Department of Defense, 1968; Ostrofsky, 1977d; Sage, 1977; U.S. Air Force, 1966, 1974; Wymore, 1976). Note, for example, that Asimow's (1962) last four design phases (Figure 2) expressly addressed what he perceived to be the four primary life cycle phases: production, distribution, operations/consumption, and retirement. Production refers to the initial creation of the system. Distribution refers to all activities necessary to deploy and/or implement the system in its intended environment. Operations refers to the performance of the primary functions for which the system was originally intended. Note that the possible substitution of the term "consumption" supports the broad definition of a system we have chosen to use: some systems are consumed rather than operated. The retirement phase of the life cycle is perhaps the most frequently neglected in terms of design. It is not surprising that many designers fail to consider the potential

consequences of the ultimate obsolescence of their system. While phased revision or perpetual modification is certainly a feasible alternative for some systems, others will require consideration of disposal or neutralization activities. Increased awareness in recent years of environmental consequences has highlighted the success or failure of various system designs in this phase. Characteristics such as biodegradability and recyclability have begun to assume significantly more attention in modern design efforts.

As with delineation of the design phases, the system life cycle phases could easily be grouped or described differently without degradation as long as the content of the issues considered remained relatively the same. However, it is interesting to note that later design schemes (Hall, 1969; Wymore, 1976; Sage, 1977) have retained the four stage life cycle described above, largely unchanged.

Influence of the Systems Approach

Boulding (1956) described the quest of General Systems Theory as the development of "a body of systematic theoretical constructs which will discuss the general relationships of the empirical world." More specifically, Johnson, Kast, and Rosenzweig (1964) stated that "the aim of systems theory for business is to develop an objective, understandable environment for decision making."

The introduction and subsequent popularity of systems theory and

the broad "systems" perspective have had significant and lasting effects on the theory and practice of management (Checkland, 1981; Cleland and King, 1972; Forrester, 1968; Johnson, Kast, and Rosenzweig, 1970; Van Gigch, 1978). The following paragraphs briefly summarize the ways in which these influences have carried over into the arena of design.

Macro vs. Micro Level Design

Even today, the topic of design tends to evoke images of detailed blueprints or wiring diagrams for a specific tangible product. The design is frequently perceived to be confined to a description of the characteristics to be embodied within the physical confines of that product. However, one of the great influences of the systems movement was the concept of defining, a priori, a system's boundaries as well as its interfaces with its environment. This seemingly simple concept highlighted the realization that a system may be defined at virtually any level. In addition, the specification of interfaces requires the recognition that any system is, itself, contained within another, larger system.

These realizations encouraged designers to define an appropriate level of focus for their design activities and to consider the interactions between system and environment. The natural consequence of these influences has frequently resulted in a much broader level of system definition than had previously been the case. This broader,

macro level perspective has facilitated the ability of the designer to ensure, for example, that not only is the tangible product accounted for, but also the required support subsystems necessary for the sustained success of the system in its intended operating environment. Such considerations have complemented the broadened awareness of the system life cycle phases.

Another offshoot of the broadened systems perspective has been an increased willingness to apply structured design methods to much larger, more interdisciplinary problems than were previously considered feasible (de Neufville and Keeney, 1974; de Neufville and Marks, 1974; Department of Defense, 1968; Sage, 1977; U.S. Air Force, 1966, 1974; Wymore, 1976). Researchers recognized, for example, that the same methodology used to design a vacuum cleaner might lend support to the effective planning of an airport or a medical service system. It is this influence which has led to the broad definition of system design we provided above.

Reductionism vs. Expansionism

Prior to the advent of systems theory, it was common practice to undertake the design of a system by first decomposing the system into its constituent subsystems and then designing or optimizing each subsystem (Nadler, 1985). The final system was then synthesized by combining the optimized subsystems. Implicitly, each subsystem was

considered independent of the others. This perspective is referred to as "reductionism" (Ackoff, 1979a).

However, Sage (1981: 76) observes that expert planning is characterized by a wholistic approach rather than a decomposed one. Closely related to the definition of system boundaries and the macro level perspective of design is the focus of systems theory on the performance of the integrated system within the context of its surrounding system environment. Rather than suboptimizing, the systems approach encourages consideration of the interaction among subsystems and is, therefore, concerned with the performance of the system as a whole (Ackoff, 1979b). This approach is referred to as "expansionism" (Ackoff, 1979a; Nadler, 1985). Subsystems are viewed in a synergistic sense with the whole being greater than the sum of its parts.

Inductive vs. Deductive Design

Another characteristic of early design methods was the reliance on creativity to visualize a specific desireable system alternative early in the design process. This envisioned system then served as the basis for an inductive approach (Nadler, 1985) to its own design. Sometimes referred to as "point design", this approach implicitly assumed that the point envisioned by the designer was the best alternative to achieve the needs of the system. Therefore, the balance of the design activities focused on the ability of the

designer to achieve that point. Simply put, this approach starts with a specific solution and then endeavors to make that solution fit the problem.

By contrast, the systems perspective encourages a deductive approach to the design effort (Nadler, 1985). This method focuses on the situational needs and then explores broader theoretical alternatives in a search for that specific implementation which best satisfies the needs. Ackoff (1979b) refers to this approach as "idealized design" in which the desired system is actually a response to the perceived needs and the specific design emerges as a means to that end.

Iterative Nature of Design

A final noteworthy development in design methodology has been the increased recognition of the iterative nature of the design process. As mentioned earlier, the purposeful activities of design are generally perceived as a sequence of decisions or specifications which proceed to the ultimate solution at increasing levels of detail. However, as the designer proceeds through this series of decisions he gains additional information and insight beyond that which he possessed earlier in the process. In some cases, this new information may suggest reconsideration of decisions made earlier in the process (Ostrofsky, 1977d). Ackoff (1979b) asserts that:

> . . . plans should be continuously revised in light of (i) their performance, (ii) unexpected problems and

> opportunities that arise, and (iii) the latest information, knowledge and understanding available, much of which is derived from the implementation process.

Consequently, the ultimate quality of the system design is enhanced if the designer returns to the earlier point in the process and reiterates through the intervening steps.

The reiteration process may be perceived as costly from a short-sighted point of view due to its impact on the design project schedule. However, it is frequently the case that both the quality of the final system and the speed with which it is successfully realized are improved when iteration is accomplished. Consequently, productive iteration is encouraged rather than discouraged. In fact, recent researchers in the design of information systems have encouraged evolutionary approaches in which the objective is to obtain a working system at the earliest possible point and then focus on the process of adjusting the system to maximize its performance based on direct observation of its success or failure within the actual operating environment (Keen, 1980; Alavi and Henderson, 1981). A corollary principle espoused by Asimow (1962) is the "Principle of Least Commitment". This principle states that no irreversible decision should be made until it absolutely must be made.

Design Decision Making

We have characterized the design process as consisting of a series of decisions which must be made (either explicitly or

implicitly) by the designer. Starr (1973) has suggested that such broad design decision making problems can be visualized as being composed of three problems: the search problem, the creation (or invention) problem, and the decision problem. Foremost among these decisions is the determination of which design alternative should be pursued to implementation: the decision problem. Regardless of the design approach used, this decision is central to the ultimate success or failure of the system. Nadler (1967) has identified the decision making process used to produce the target system as one of the primary research issues in the area of system design methodology. It is this decision making process which serves as the focus of this research. Therefore, we will subsequently refer to this process simply as design decision making. In the following paragraphs, we highlight the characteristics of the design decision process.

Complex Decision Environment

The success or quality of design alternatives in most practical environments (particularly for large-scale systems) typically involves consideration of multiple objectives (Easton, 1973a). Unfortunately, it is common for the optimization of some characteristics to run counter to the realization of other system design objectives (e.g. cost vs. schedule). Consequently, the design environment is typically characterized by the need to simultaneously satisfy multiple conflicting objectives.

It is also true that today's design environment is characterized by a high degree of complexity. Rapid advances in computing and communications have permitted the inclusion of sophisticated feedback and control systems in even the simplest of modern products and systems. This has meant not only that greater technical complexity is present in design alternatives, but also that the number of alternatives available in the design/planning process has increased exponentially with the possible combinations of hardware, software, and procedures (Easton, 1973b; Starr and Greenwood, 1977).

The planning and psychology literature clearly substantiate the difficulty of the human mind in dealing effectively and consistently with such complexity, conflicting objectives, and large numbers of alternatives (Bowman, 1963; Brunswik, 1947; Dawes, 1971; Dawes and Corrigan, 1974; Ebert and Kruse, 1978; Goldberg, 1970, 1976; Hammond, 1955; Hamner and Carter, 1975; Hogarth and Makridakis, 1981a, 1981b; Hurst and McNamara, 1967; Johnson and Payne, 1985; Kunreuther, 1969; Todd, 1954; Tversky and Kahneman, 1974). Consequently, there is a continuing need for usable, yet objective decision aids in the design/planning process.

Prescriptive vs. Descriptive Decision Models

Decision making models are typically either descriptive or prescriptive in nature. Descriptive models attempt to describe or

explain how decisions are currently made. Prescriptive models, on the other hand, attempt to explain how decisions should be made.

Descriptive decision models are predominant in fields such as psychology and management information systems. The objective in these areas is to understand not just what decisions are made in a given situation, but also to peer inside the "black box" of the decision process to understand why a particular decision is reached (Todd and Benbasat, 1987). The rationale for these models lies in the belief that if we understand the strengths and weaknesses of the human decision process we may be able to accentuate the strengths and counter the weaknesses in such a way that the quality of the resulting decisions is enhanced. These models frequently include consideration of factors such as cognitive style and communication patterns in addition to raw problem data.

By contrast, prescriptive decision models rarely attempt to mirror the actual human decision process. Instead, the objective of these models is to be able to take the same input pieces of information and produce an output decision which is better than that which would have been achieved using other methods. The focus is on the quality of results, not on the method of achieving those results. Prescriptive models are typically quantitative in nature. It is this form of decision making model which will be pursued in this research.

Subjective and Objective Considerations

Most complex decisions involve both subjective and objective considerations. Structured design methodologies and prescriptive decision models generally focus on objective factors almost to the exclusion of subjective ones. Yet in the absence of such structure, we typically observe that humans give subjective factors very high priority. This may be partially explained by the perceived difficulty of making objective assessments. However, another explanation may lie in the recognition that subjective factors, while perhaps not as amenable to quantification, are nonetheless perceived as crucial ingredients in the decision making process. Ackoff (1979a) persuasively argues the narrow-mindedness and ultimate futility of attempting to restrict our decision making considerations to solely objective judgments:

> Objectivity is _not_ the absence of value judgments in purposeful behaviour. It is the social product of an open interaction of a wide variety of subjective value judgments. _Objectivity is a systematic property of science taken as a whole, not a property of individual researchers or research_. It is obtained only when all possible values have been taken into account; hence, like certainty, it is an ideal that science can continually approach but never attain. That which is true works and it works whatever the values of those who put it to work. It is _value-full_, not value-free.

Therefore, the ideal decision model should provide the structure to consistently include both objective and subjective considerations (Hall, 1962).

Relative Importance of Criteria

We have previously described the design decision environment as being characterized by multiple (typically conflicting) objectives. However, it is possible and, in fact, quite likely that not all decision criteria will be equal in importance. Therefore, one challenge to be met by decision models is the need to treat multiple criteria of unequal importance. This weighting process may well be one of the ways in which subjective considerations enter the decision process.

An additional complication in the treatment of relative weights is the recognition that the relative importance of criteria may vary as the performance of a system varies. In the context of clinician judgement, Hoffman (1960) has observed that:

> . . . clinicians seldom believe that linear functions best describe relationships between information and characteristics being judged. It may, in fact, be closer to the truth that, at least for some classes of information, extreme scores are more decisive in judgement than are scores in the middle range . . . For some (clinicians), scores above or below some arbitrary value may carry no added significance whatsoever.

More directly related to the context of design, Shestakov (1983) identified several examples where the decision maker's preferences may change when performance measures cross some threshold value, i.e. preferences may vary within intervals.

As the number of preference intervals approaches infinity or the ability to identify discrete interval breaks diminishes, the concept

of varying relative weights may be seen to evolve toward approximation by continuous, perhaps even nonlinear, functions. In the business or management context, cost considerations may take on increasing importance relative to other design criteria as the cost of a particular alternative approaches budgetary constraints. Conversely, while some minimum level of safety may be considered crucially important, additional levels of safety may be perceived as yielding diminishing returns.

Satisficing vs. Optimization

From a descriptive perspective, Simon (1955; 1958) has suggested that the decision process typically involves the evaluation of alternatives with respect to some minimum standard of performance. Decision making is characterized as involving a search for alternatives which meet this minimum desired level of performance: solutions which are "good enough". This concept is referred to as satisficing.

By contrast, prescriptive decision models typically attempt to identify the alternative whose performance is expected to be the best possible with respect to the design criteria. This goal is referred to as optimization.

Folkeson (1982) suggests that satisficing may be perceived as a practical form of optimization in which rational trade offs are made

between the complexity of the decision process and the importance of the potential outcome. The more valuable the desired outcome, the greater the motivation to seek optimization. While this idea seems perfectly valid, it still infers that large-scale design efforts with far-reaching consequences merit the pursuit of optimality.

However, given the complications of multiple conflicting objectives and the presence of subjective considerations, it is clearly debateable whether provable optimality is a realistic objective. Therefore, Keen (1977) has observed that optimality in such situations is an evolving concept which eludes absolute definition, depending instead on the context of the problem and the needs and ability of the decision maker. Consequently, we are forced to base our definition of optimality on selection of an appropriate theory and subsequent faithful implementation of that theory. Using such an approach, we hope to emerge with a decision which was logically derived and is reasonably defensible within the context of our problem, despite our inability to assert that it is indisputably and unchangeably the "ideal" solution.

Interaction Among Criteria

As discussed earlier, design activities prior to the advent of systems thinking were frequently undertaken using a reductionist perspective in which subsystems were optimized under the assumption of independence. While the expansionist perspective of systems theory

has helped combat this problem in terms of the relationships among the subsystems being designed, a parallel problem may be seen to exist with respect to the multiple design criteria. Specifically, it is quite reasonable to expect that factors which influence the performance of a design alternative on one criterion may also affect the performance of that alternative on some other criterion. Therefore, any decision making approach which treats design criteria as independent may fail to recognize the effect of interactions among criteria, thus increasing the risk of an improper optimal choice. Consequently, Hoffman (1960) advises that:

> The inclusion of interaction terms in a (decision) model takes account of the possibility that for a particular judge the interpretation of one item of information may be contingent upon a second . . . it is possible to include terms involving higher order interactions or other postulated functional relationships among the information variables.

Objectives of This Research

The issues discussed in the preceding paragraphs support Nadler's (1967) promotion of the need for continued research in design decision making. This need has been repeated more recently (Rabins et al, 1986; Tompkins, 1989) and is referred to by Starr (1973) as simply "the decision problem". Using these issues as a guideline, we define the objective of this research to be <u>the development of a prescriptive design decision making approach</u> which:

1) supports comparative evaluation of a large number of design alternatives,

2) considers multiple, potentially conflicting, objectives,

3) considers both objective and subjective relevant factors,

4) permits relative weights which vary both among criteria and within the range of possible criteria values,

5) seeks decisions which, within the context of the defined problem, are optimal in the sense suggested by Keen (1977) and discussed earlier in this chapter, and

6) considers interactions among criteria.

In achieving this objective, the research will contribute a method which responds to decision making needs in the design of new, increasingly complex systems. In addition, the method developed will provide a significantly more realistic and effective means for resolving previously defined problems.

Organization

Chapter 2 facilitates examination of the decision process by describing a specific formal design methodology which provides a suitable framework for design decision making. Chapter 3 contains a review of significant existing prescriptive approaches to the decision making process, an examination of their respective and collective strengths and weaknesses in the design context, and the selection of the method which serves as the basis for development of the desired approach. Chapter 4 provides the theoretical development of the desired approach. Chapter 5 contains a demonstration of the suggested

approach via application to an actual problem. Finally, Chapter 6 summarizes, including a review of the achievement of the research objectives, an assessment of the limitations of the research, and suggested important areas for future research.

Chapter 2

DESIGN/PLANNING METHODOLOGY

Johnson, Kast, and Rosenzweig (1964) have observed that "if the system within which managers make the decisions can be provided as an explicit framework, then such decision making should be easier to handle." Therefore, to facilitate the development of the decision making process which satisfies the stated research objective, it is first necessary to establish the context of a design methodology which implements the significant methodological developments described earlier. It can be argued that many approaches meet this requirement. While the names and sequence of steps differ from one approach to another, most contain only slight variations of the same basic principles. Since the focus of this research is on the decision making process, it is sufficient for our purposes simply to identify a background methodology which possesses the desired characteristics rather than attempting to identify the single best methodology. Therefore, this chapter describes such a methodology and demonstrates its embodiment of the desired features without asserting that it is the only such approach.

The design morphology formulated by Asimow (1962) was based

simply on his observations of a sequence of decisions which must be made (either explicitly or implicitly) in any design/planning process. Ostrofsky (1977d) later developed and extended Asimow's design morphology to its present form (Figure 3).

The stages in Figure 3 are broadly grouped into two classifications based on the system life cycle concept: the Design/Planning phases and the Production/Consumption phases. The former include those phases which span the activities from the conceptualization of the system until its implementation and constitute the design methodology itself. The last four phases represent the system life cycle subsequent to the design activities and represent the target activities of the design effort. Each of the steps included in the Design/Planning phases must be accomplished, largely in the sequence shown. This is not to say that each step is final. On the contrary, the discovery of additional information or the determination that previous steps were inadequately accomplished is intended to result in reiteration through as many of the preceding steps as necessary.

The focus of this research will be the decision process embedded in the Preliminary Activities phase. However, in order to adequately address this decision process it is necessary to first establish the relevant context by briefly describing the complete set of Design/Planning phases. The following paragraphs provide the needed

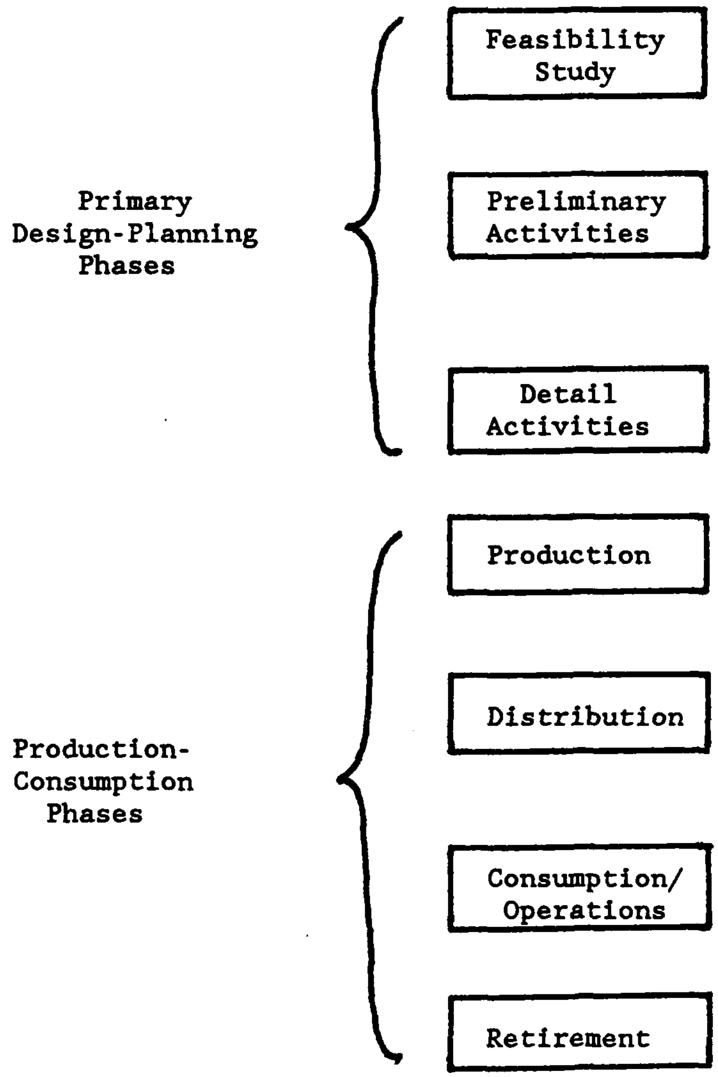


Figure 3. Asimow/Ostrofsky Design Methodology

(Ostrofsky, 1977d)

description, drawing heavily on the original work of Asimow (1962) and Ostrofsky (1977d).

Feasibility Study

The purpose of the Feasibility Study is to establish a set of solutions to meet the stated needs. Note that this goal implies both a statement of needs and generation of potential solutions. The specific steps required are shown in Figure 4. Each of the indicated steps is described in the following paragraphs. Figure 4 formally reflects the presence of iteration when new information or insight indicates the need to revisit previous decisions.

Needs Analysis

In this first crucial step, a general statement of the problem is agreed on and documented. This requirement serves not only to define the direction of subsequent activities, but also to verify that there appears to be sufficient need to justify a serious pursuit of a solution.

Typically, some documentation concerning the need has been created prior to initiation of the design effort. While this is certainly an important input to the needs analysis, it is not the only input. Experience indicates that written statements virtually never contain a complete description of the needs. Therefore, it is

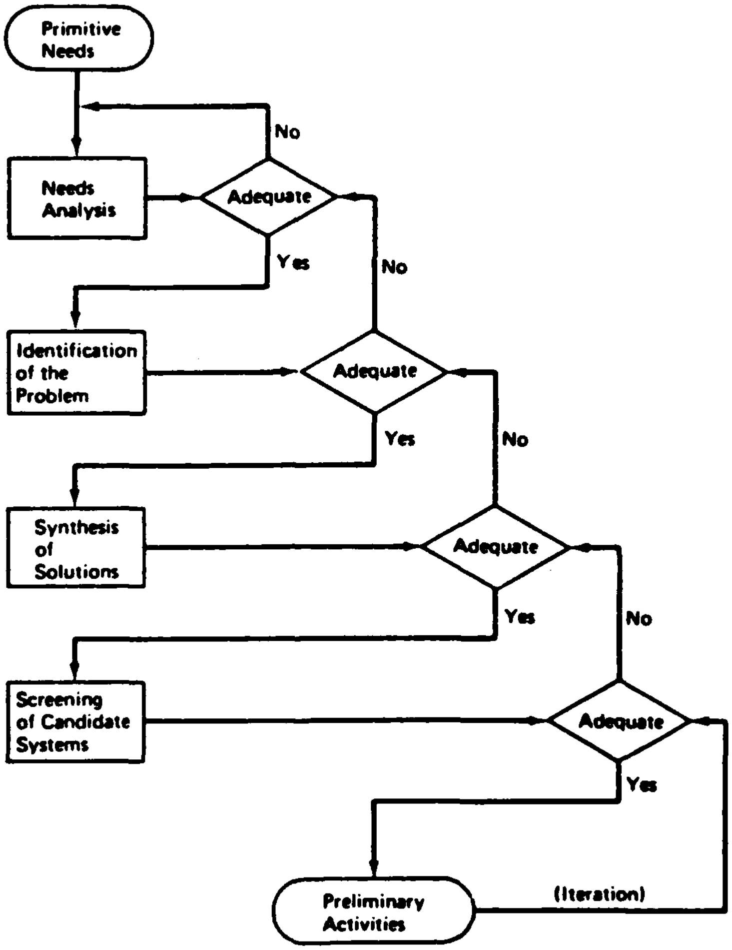


Figure 4. The Feasibility Study

(Ostrofsky, 1977d)

suggested that observation be used as a supplemental source of information. This relationship is shown in Figure 5.

The process of examining and stating the needs forces those involved to evaluate the urgency of the needs. The result of this evaluation should then be compared to the potential cost in time and resources necessary to complete a formal in-depth design effort. In other words, the perceived worth of the resulting system must be at least as great as the costs necessary to design it. For simpler systems this coarse screen may indicate that the magnitude of activities we will describe is not merited. However, for large-scale, complex systems this will virtually never be the case.

Identification and Formulation of the Problem

While the broad problem statement emerging from the Needs Analysis provides a crucial starting point for the design effort, it generally lacks the specific detail necessary for the subsequent design actions. If activities proceed without further clarification, the risk exists that the solution may not be responsive to the true needs. Therefore, this step strives to refine the problem statement to a greater level of detail.

The framework for this effort is formed by combining the system inputs/outputs with the system life cycle (Production/Consumption) phases to form a two-dimensional matrix whose cells require

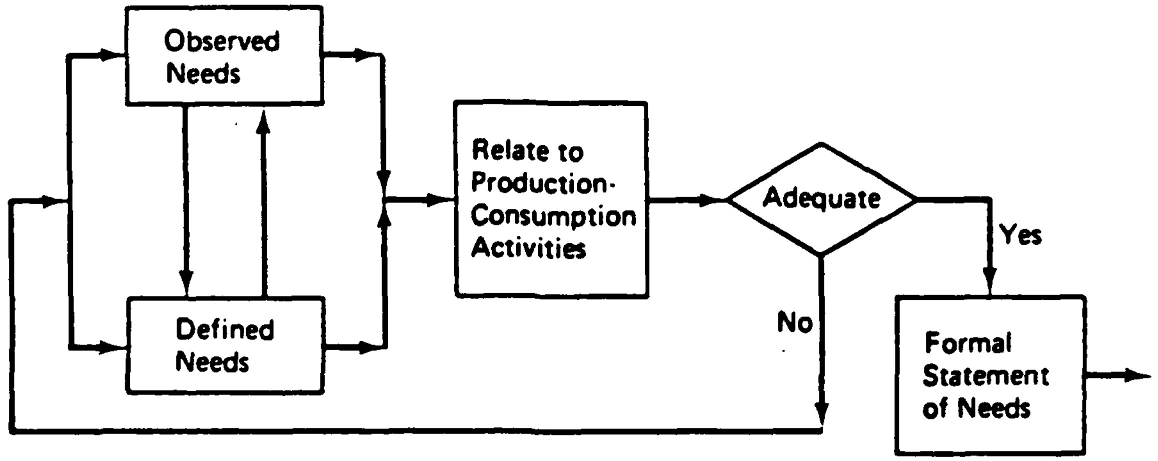


Figure 5. Needs Analysis

(Ostrofsky, 1977d)

specification. Recall that the system inputs/outputs defined by Asimow (1962) are the environmental inputs, intended inputs, desired outputs, and undesired outputs. Further recall the definition of four life cycle phases subsequent to the design activity: production, distribution, operations/consumption, and retirement. Consequently, identification and formulation of the problem produces a sixteen cell matrix as shown in Figure 6. This is referred to as the input-output matrix.

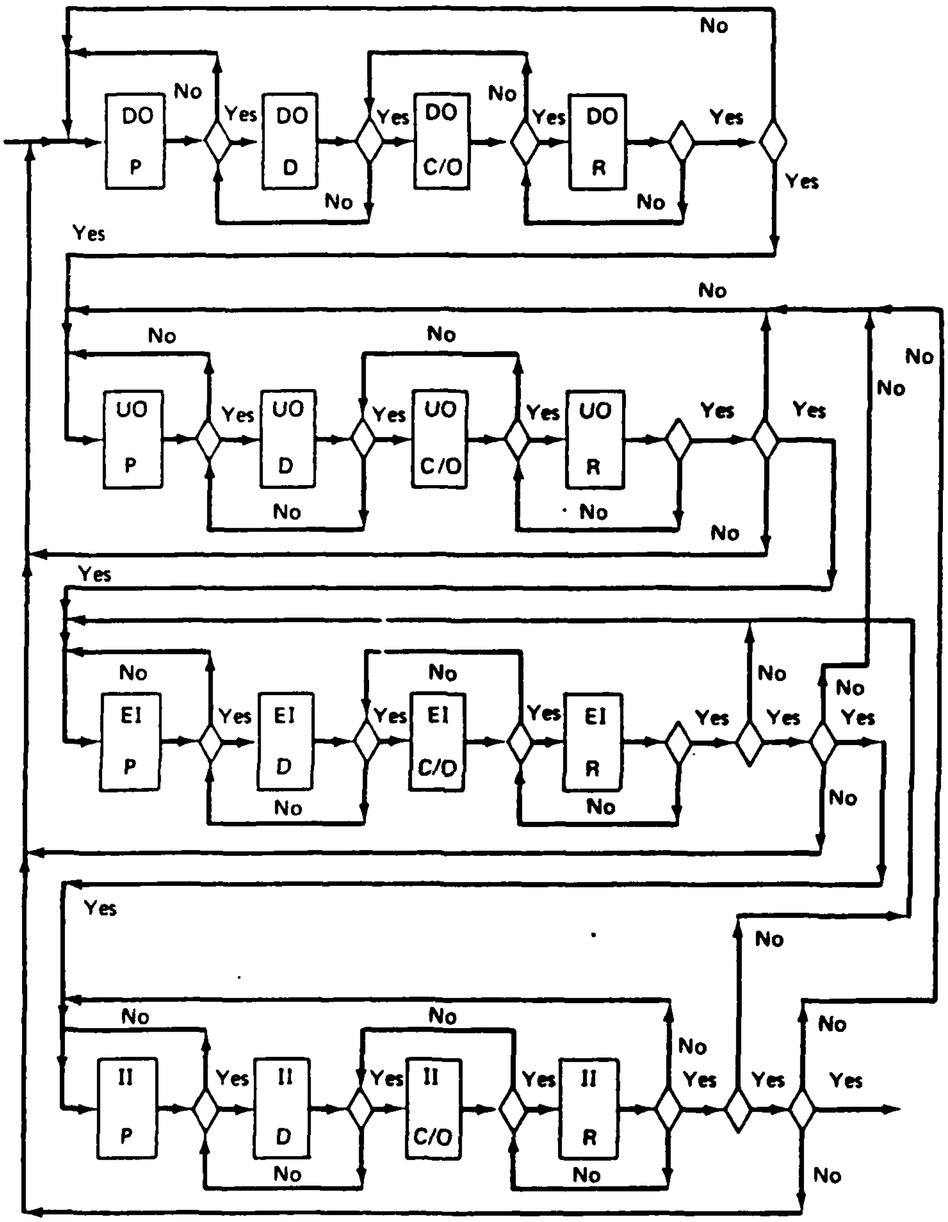
Although it is theoretically possible to complete the matrix cells in virtually any order, experience and logic have produced the recommended completion sequence shown in Figure 7. In this sequence, one would first specify the desired and undesired outputs of each phase of the system life cycle. Next, the environmental inputs (i.e. those inputs to the system which will exist without any intervening efforts) for each phase are defined. Finally, having specified what is desired from each phase and what inputs (both good and bad) will be available, it remains only to specify the missing input ingredients which must be made available to generate the desired outputs.

Take as a simple example the planning of a concert to be held in a public park. A primary desired output in the consumption/operations phase might be an entertained and satisfied audience. An undesired output might be disturbed residents in the vicinity of the park. Environmental inputs might include the weather and any existing park facilities. Based on the inputs and outputs specified so far, it is

	Inputs		Outputs	
	Intended	Environmental	Desired	Undesired
Production				
Distribution				
Consumption-Operation				
Retirement				

Figure 6. Input-Output Matrix

(Ostrofsky, 1977d)



DO = Desired outputs
UO = Undesired outputs
EI = Environmental inputs
II = Intended inputs

P = Production
D = Distribution
C/O = Consumption/operations
R = Retirement

Figure 7. Sequence for Input-Output Matrix Completion

(Ostrofsky, 1977d)

clear that factors such as a band, additional facilities, and advertising might be specified as intended inputs. With respect to the retirement phase, desired outputs might include safe and efficient transportation home for the participants. Litter might be an undesired output. Prevailing traffic conditions and capacities might be an important environmental input. Finally, traffic directors and cleanup crews might be perceived as necessary (intended) inputs to the concert's retirement phase. The production and distribution phases are described in a similar manner.

Synthesis of Solutions

The preceding steps of the Feasibility Study were analytic in nature since they began with broad concepts and attempted to break them down into meaningful components. The result of these efforts is a detailed formulation of the problem to be addressed. The remaining two steps of the Feasibility Study attempt to respond to these needs by the complementary process of synthesizing alternative solutions. While analysis separates an entity into its constituent elements, synthesis is the opposite process in which separate elements are combined into a unified entity. The purpose of the Synthesis of Solutions is to generate as many potential design alternatives (candidate systems) as possible to meet the defined needs by combining various approaches to performing each required system function.

Given this objective, virtually any creative technique which

encourages generation of alternatives may be employed. However, experience has shown the following approach to be useful for this purpose, both because it provides a well-defined, perhaps even mechanical aid to creativity and because the process helps insure thorough consideration of the possible alternatives.

Synthesis is accomplished by first identifying broad approaches which accomplish each function required of the final system. These approaches are referred to as "concepts". It is generally the case that more than one concept can be identified to meet the needs. For example, if the stated need is to establish some means of passenger transportation between the east and west coasts of the United States, one concept might be an airplane, another might be a bus, and a third might be a train.

Next, each concept is subdivided into the various subsystems necessary for it to meet the needs. For example, the train concept might be subdivided into the following subsystems: passenger terminal, passenger seating, dining facilities, sleeping facilities, propulsion (i.e. locomotive), etc. It is generally the case that there will be more than one means of achieving each subsystem. Therefore, the alternatives are identified for each subsystem of each concept as shown in Figure 8. Synthesis of candidate systems then becomes a straightforward matter of forming all possible combinations of subsystem alternatives. If a concept has three subsystems, with the subsystems having five, four, and eight alternatives respectively,

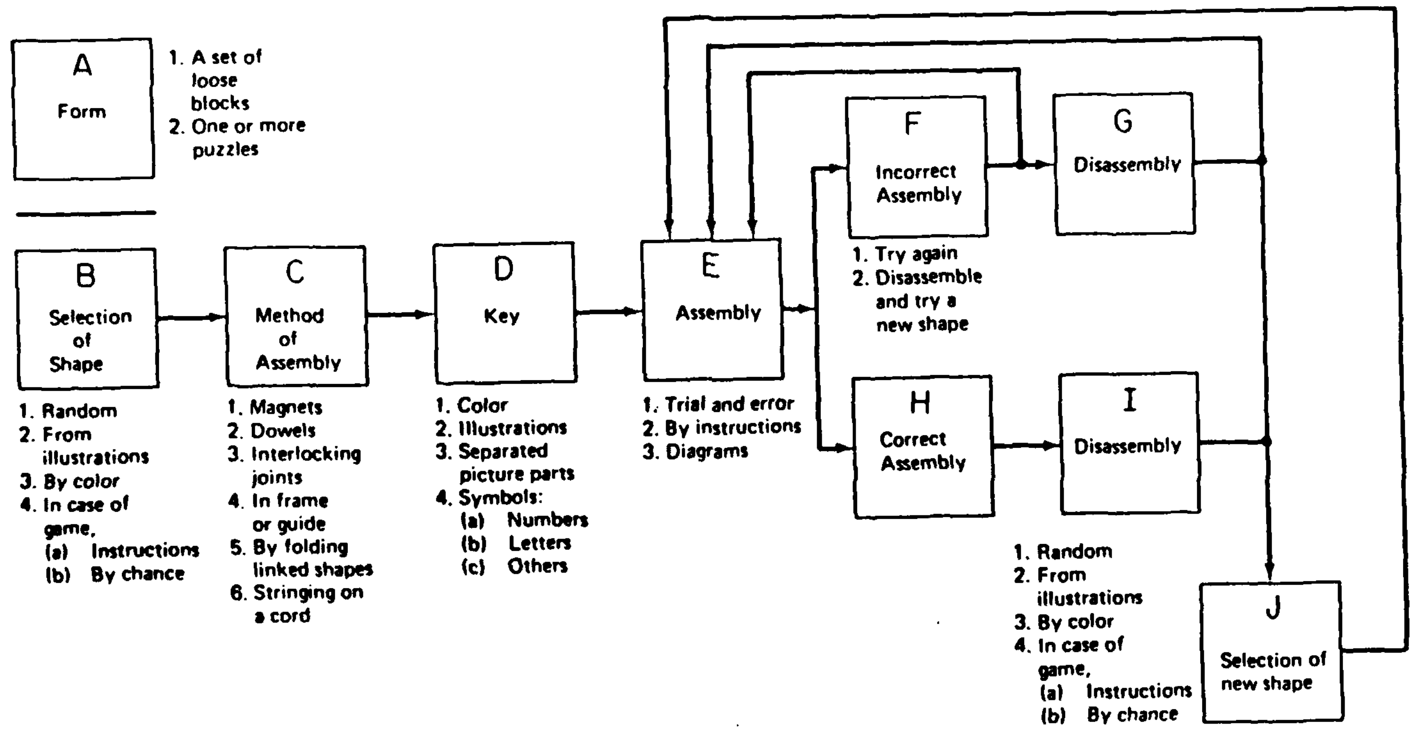


Figure 8. Concept Subsystem Alternatives

(Ostrofsky, 1977d)

then a total of 160 candidate systems (5x4x8=160) may be synthesized. This process is repeated for each concept to form the total pool of candidate systems. (For the remainder of this paper we will use the subscript α to represent a specific candidate system.) Howard (1988) refers to the representation of this approach to the specification of alternatives as the "strategy generation table".

At this point in the design process, the focus is on producing the largest possible pool of candidate systems. The reason for this objective lies in the concept of optimization. Simply put, if we speculate that for each given design problem there exists some theoretically best (optimum) candidate system, then by increasing the number of candidate systems we identify, we may increase our chances of considering the optimum solution. Unfortunately, since we lack an absolute benchmark for optimality and since we can never be sure we have considered every possible candidate system, we will never know if we have selected or even considered the optimum solution. Therefore, we will subsequently differentiate by using the term "optimal" to refer to the candidate system we select.

It is likely that some of the candidate systems generated in this manner may be unfeasible or undesirable. While we do not wish to immediately eliminate any candidates during the synthesis process for the reason just described, the next step reduces the magnitude of evaluating alternatives by attempting to eliminate such alternatives from further consideration.

Screening of Candidate Systems

While we have attempted to synthesize as many candidate systems as possible, we do not wish to waste resources evaluating alternatives which are clearly unachievable or undesirable. Therefore, in this step we establish three screens (Figure 9) to eliminate those candidate systems which do not merit further evaluation.

The first screen is that of physical realizability. Candidate systems which cannot physically be implemented are eliminated. For example, if one subsystem of a concept requires availability of an electric power source then all candidate systems formed by pairing that alternative with non-electric power sources might be deemed physically unachievable and eliminated. Recognizing that large design efforts are likely to span long periods of time, this determination is not made based solely on current capabilities, but also under consideration of capabilities which <u>might be achieved within the planning horizon</u>.

The second screen concerns economic worthwhileness. This screen eliminates all candidate systems whose potential value clearly does not exceed the resources necessary to achieve it. Traditional techniques such as breakeven analysis may be useful in this regard. Note, however, that the determination of value may be made in terms of any measures deemed appropriate and is not necessarily restricted to

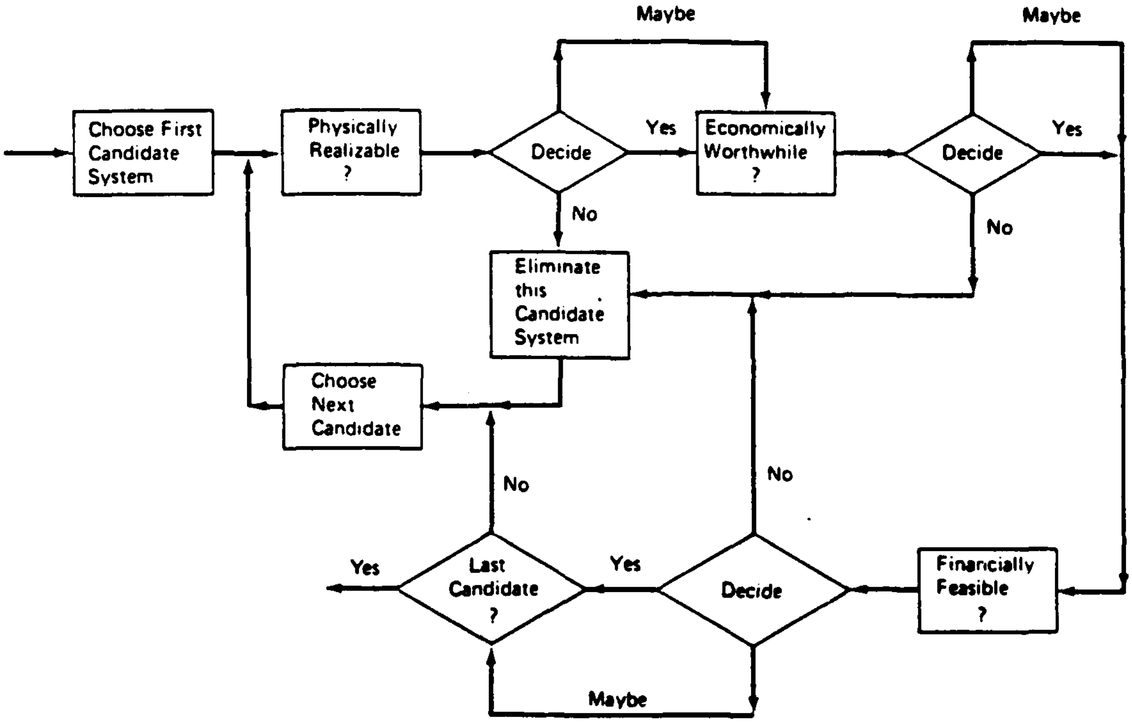


Figure 9. Screening of Candidate Systems

(Ostrofsky, 1977d)

monetary considerations. The objective is simply to establish rough preliminary assessments of the tradeoffs between the costs of implementation and the benefits to be derived. While this examination is similar to the tradeoff evaluation implicitly generated initially in the Needs Analysis, it goes much farther. The earlier assessment would only have discouraged continuation if the projected benefits could not even exceed the cost of the design effort itself. Here, a more selective evaluation is made with respect to individual candidate systems and the potential costs considered include all those which would accrue throughout the system's life cycle.

The third screen concerns financial feasibility. This screen simply asks whether the minimum resources necessary to achieve the candidate system are or could become available. If not, the candidate is eliminated from consideration.

At this early stage of the design process, little factual data may be available concerning the candidate systems. Therefore, the physical, economic, and financial screens are likely to be fraught with uncertainty. To govern the decision in such cases, the pervasive rule is established that <u>no candidate system is eliminated unless it is clearly unachievable, uneconomical, or infeasible</u>. In other words, if in doubt, the candidate system should not be eliminated in order to retain the highest possibility that the optimum has been included. The tradeoff between the desire to find the optimum by maximizing the size of the set of candidate systems and the corresponding difficulty

in thoroughly evaluating a large set of alternatives is referred to as the "designer's dilemma."

Preliminary Activities

Using the set of candidate systems synthesized during the Feasibility Study, the purpose of the Preliminary Activities becomes the selection of the optimal candidate system: that is, the defined candidate system which will best satisfy the stated needs.

Preparation for Analysis

Since large-scale design efforts frequently span a long period of time, it is not unusual for temporal breaks to occur between major portions of the design activities. Such breaks may occur to permit formal reviews, funding decisions, or organizational and staffing efforts for the more resource-intensive efforts to follow. The proposed methodology recognizes the need for such breaks by including a preparation step at the start of the second major phase. The purpose of this step is to provide an opportunity for designers to refamiliarize themselves with the results of the activities which have gone before and adjust to changes required by improvements revealed subsequently to original decisions. Although the rigor with which this step is accomplished will vary based on the nature of the effort and the time which has elapsed since the Feasibility Study, it

provides a logical basis for educating new project personnel as the design effort gradually expands beyond the smaller cadre typically involved in the early activities.

Since the primary focus of this step is on familiarization, the recommended approach is to reexamine the synthesized set of candidate systems, grouping them according to their common characteristics. The input-output matrix serves as a basis for this grouping. Although there is no formal output from this step, the increased awareness of the characteristics of the candidate systems and the stated needs serves as a basis for subsequent analytic activities. It is also possible that the process may highlight the need for iteration, especially if a substantial period of time has elapsed since the completion of the Feasibility Study.

Definition of Criteria and Relative Weights

Since the purpose of the Preliminary Activities is the selection of the optimal candidate system, it is necessary that decision criteria be established. Simply stated, criteria are the standards against which the performance of the system will be evaluated. Criteria are normally characteristics which are broad enough to preclude direct measurement, e.g. safety, availability, performance, and even cost. Although there is typically broad implicit agreement on the relevant criteria, their formal statement is essential since quantitative mechanisms will be used for the evaluation process.

Therefore, any criterion not expressly considered will not be included in the selection of the optimal candidate system.

At this stage there may be a temptation to specify each detailed characteristic that the envisioned system should have. This temptation should be resisted, however, since subsequent steps in the methodology will be used to "flesh out" the relevant details. Instead, this step is intended to highlight the broad inclusive qualities desired.

Once a set of criteria (denoted $\{x_i\}$) has been established, each criterion is assigned a relative weight (denoted $\{a_i\}$). This action is based on the generally applicable case where not all criteria are of equal importance. For reasons which will be shown later, the relative weights are defined such that

$$\sum_{i=1}^{n} a_i = 0 \qquad \text{Eq. 2.1}$$

where the subscript i refers to the ith criterion.

A variety of methods have been established in the literature to facilitate the derivation of relative weights (Churchman, 1957; Fishburn, 1964; Huber, 1974b). A comparison of these approaches is beyond the scope and intent of this research. For our purposes, any method is acceptable which yields weights reflecting the desired objectives of designers, producers, users, etc. Given the

multiplicity of relevant parties, a method which explicitly accommodates group preferences (e.g. Delphi, group decision support systems, etc.) would normally be desirable. In the simplest case of constant relative weights, the resulting description of criteria might look like the example in Table 1.

Definition of Parameters

While the previous step identified the broad characteristics of interest, many of these attributes will not be directly measurable. Therefore, in this step, directly measurable characteristics (parameters) are identified from the set of candidate systems which can later be combined to represent the defined criteria. This process is achieved as follows.

For each criterion, a brainstorming effort is undertaken to identify an exhaustive set of constituent elements for that criterion. These elements are then listed in tabular form and coded (Table 2) to indicate whether each element can be:

a) directly measured,

b) measured from some model that includes other elements coded as a's or b's,

c) completely included in other elements, or

d) not measurable within existing resources (or more accurately,

TABLE 1.

CRITERIA WITH RELATIVE WEIGHTS

(Ostrofsky, 1977d)

Criterion, x_i	Weight, a_i
x_1, play value	$a_1 = 4$, $4/20 = .20$
x_2, educational value	$a_2 = 3$, $3/20 = .15$
x_3, production cost	$a_3 = 5$, $5/20 = .25$
x_4, durability	$a_4 = 2$, $2/20 = .10$
x_5, quality	$a_5 = \underline{6}$, $6/20 = \underline{.30}$
	20 1.00

TABLE 2.

CRITERION ELEMENTS

(Ostrofsky, 1977d)

Criterion: Play value

Elements	Code
Enjoyment	b
Interest	b
Number of colors	a
Number of moving parts	a
Number of instructions required to operate	a
Assembly time	a
Operating time	a
Style	b
Density	b
Number of different types of surface	a
Toy volume	a
Toy weight	a
Number of ways of moving	a
Internal volume	a
Fascination	d

resources planned to be available within the timeframe of the design effort).

The goal of this process is to be able to define all constituent elements as a, b, or c so that they may be included in the evaluation process. If items are coded as d, they must be individually examined to see if the importance of their consideration merits additional efforts to obtain their measurement, even using subjective scales or proxy attributes (Keeney and Raiffa, 1976: 55) if necessary.

Since elements coded as c (completely included in other elements) need not be explicitly considered any further, the task remains to relate those elements coded as a and b. Recall that these designations indicate that the elements can be directly measured or measured as a model of the other elements, respectively. The elements coded "a" are termed parameters (denoted $\{y_k\}$) and the "b" elements are designated as submodels (denoted $\{z_j\}$). A table such as Table 3 is constructed to relate the parameters and submodels to each other. Note that a submodel will typically be measured from more the one parameter and that parameters may be included as inputs to more than one submodel. Also note that submodels may serve as the inputs to other submodels.

Given the relationships between parameters and submodels, it remains to relate both to their parent criteria. This process may be facilitated using a table such as that shown in Table 4. Note that

TABLE 3.

RELATION OF PARAMETERS TO SUBMODELS

(Ostrofsky, 1977d)

Parameters, y_k (a)	Submodels, z_j (b): Enjoyment	Interest	Style	Density
Number of colors	X	X	X	
Number of moving parts	X	X		
Number of instructions required to operate	X	X		
Assembly time	X	X		
Operating time	X	X		
Number of different types of surface	X	X	X	
Total volume	X	X		X
Total weight	X			X
Number of ways of moving	X	X		
Internal volume				X

TABLE 4.

RELATION OF PARAMETERS AND SUBMODELS TO CRITERIA

(Ostrofsky, 1977d)

Parameters, y_k (a)	Enjoyment	Interest	Style	Density	Play Value	# Lessons	Cost moving	Cost nonmoving	Cost assembe	Pkg & Overhead	Prod Time	Compress Strg	Tensile Strgth	Sheer Strength	Prob Survival
Criteria, x_i	x_1				x_2		x_3					x_4			
Submodels, z_j (b)															
Number of colors	X	X	X		X										
Number of moving parts	X	X			X		X		X		X				X
Number of instructions required to operate	X	X			X	X									
Assembly time	X	X			X				X	X	X				
Operating time	X	X			X	X									X
Number of different types of surface	X	X	X		X		X	X							X
Total volume	X	X		X	X					X	X	X	X	X	X
Total weight	X			X	X					X	X	X	X	X	
Number of ways of moving	X	X			X										
Internal volume				X	X					X					
Number of lessons covered	X				X	X									
Number of toys produced							X		X	X	X				
Cost of material per toy							X			X	X				
Number of nonmoving parts	X	X			X			X	X	X	X				X
Part thickness												X	X	X	
Temperature allowed					X							X	X	X	X

this format highlights the possibility of parameters relating not only to more than one submodel, but also to more than one criterion. Therefore, it is entirely plausible that a submodel for one criterion is based on parameters relevant to a number of other criteria. It is also important to note that there is no limit to the number of levels of submodels.

During this identification process, numerous opportunities for error exist. Therefore, three formal tests have been identified: completeness, compactness, and consistency (Figure 10).

The test for completeness requires that each criterion be examined to ensure that <u>all</u> parameters and submodels required for its evaluation have been identified. As is the case with the criteria themselves, any element not considered will not be included in the determination of the optimal candidate system.

Checking for compactness refers to insuring that <u>only those parameters and submodels necessary</u> have been included. Since data will have to be gathered from every candidate system for each parameter, the size of the parameter set is obviously related to the time, effort, and cost of the evaluation process. Therefore, reasonable opportunities to economize the set of parameters should be pursued. For example, in some cases, a direct measurement may exist which can eliminate the need for multiple parameters (e.g. pressure vs. force and area).

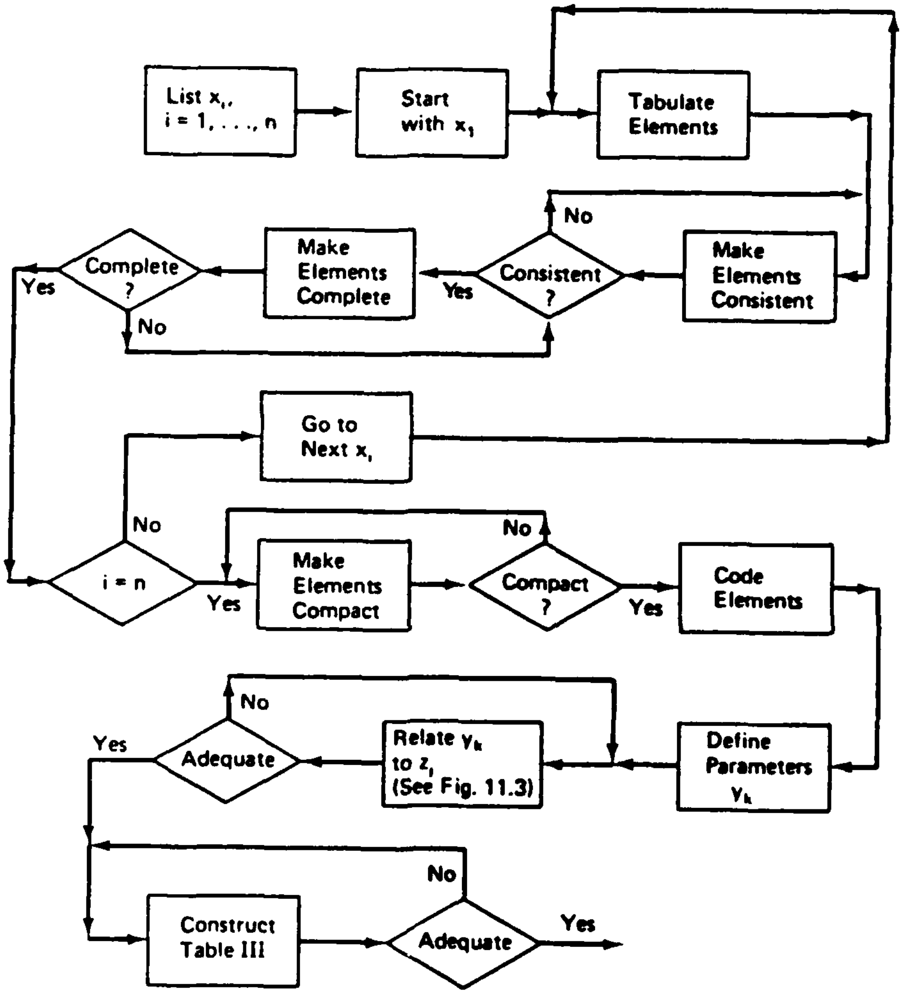


Figure 10. Tests of Criteria Elements

(Ostrofsky, 1977d)

Finally, testing for consistency means checking to insure that a parameter (or submodel) has the same meaning each time it is used. Frequently, synonyms or closely related characteristics may have initially been identified as necessary for the determination of a particular criterion when, in reality, a different attribute is really required. Table 4 provides a handy means of highlighting such discrepancies.

Criterion Modeling

This step links parameters, submodels, and criteria by establishing quantitative models which relate them to each other. In many cases, theoretically suggested and/or practically established relationships may exist (e.g. inventory models, physical relationships, experimental observation, etc.). In other cases, a more subjective process of establishing relationships must be undertaken. In the latter case, it is typically useful to begin by establishing functional relationships such as those shown in Figure 11. Given the general nature of the perceived relationship, it becomes an easier (though not necessarily easy) process to establish specific relationships via trial and error or sensitivity analysis. These relationships may require field or laboratory testing for their initial determination or verification.

In the case where established quantitative relationships are

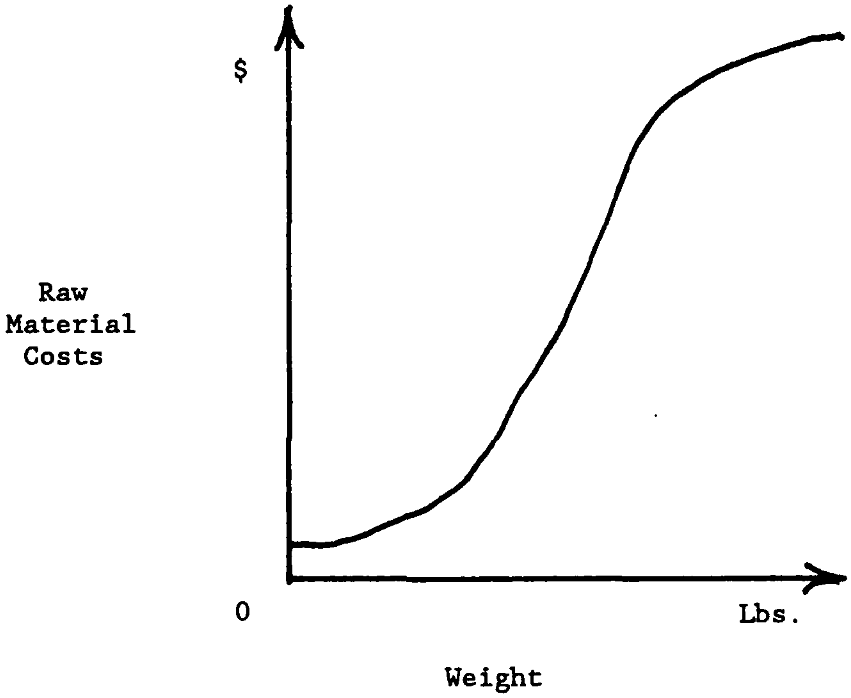


Figure 11. Relationship Between Parameter "Weight"
and Submodel "Raw Material Costs"

unavailable, there may be reluctance to speculate on the nature of those relationships. The analyst then assumes the task of reemphasizing that factors not considered will not be included in the selected candidate system. If a characteristic is important enough to be considered it must be formulated to the best extent practical even to the development of field tests to provide data. This decision defines the meeting ground between theory and practice and is precisely the issue which differentiates management from pure science. Whereas the scientist may confine himself to factors which are provable, the manager must do the best he or she can with the resources available. A useful source of encouragement in this regard is the consideration of the impact of <u>not</u> specifying such a relationship. It is also worthwhile noting that in the absence of a quantitative representation managers are still likely to consider important factors, but in an even less structured and more subjective manner. Therefore, the goal becomes that of applying as much objectivity as possible to an inherently subjective situation (Keeney and Raiffa, 1976: 45). Formal consideration of such relationships at least makes the nature of the consideration explicit, forces discussion, and facilitates revision if necessary.

The resulting models should be constructed such that criterion values increase with higher numerical values. In other words, the higher the value of the model output, the better the performance of the candidate system. Clearly, however, the output of each model will likely be in units of measure not compatible with those of other

models. Therefore, it remains for the decision model to provide a means of combining or comparing those outputs.

Formulation of the Criterion Function (CF)

Simon (1977) defines a criterion function as "a measure to be used for comparing the relative merits of various possible courses of action." In this step of the methodology, the criteria measures generated above are weighted and combined with each other in a manner consistent with the values of the design decision maker(s). This process requires an evaluation of the possible ranges of values, normalization of the criterion values, and integration of the relative weights.

It is likely that each candidate system may be capable of assuming any value within a range for some parameters. Therefore, a determination is made of the minimum and maximum values for each parameter for each candidate system. Overall parameter ranges are then determined by taking the minimums and maximums over the complete set of candidate systems. In addition, user-defined constraints may be established for parameter value ranges. It may be necessary, for example, to ensure that various parameter values do not exceed or fall below certain levels. These constraints are referred to as "regional constraints". Constraints which result from physical or logical relationships (i.e. via submodels) are termed "functional

constraints". Candidate systems whose parameter ranges violate regional constraints may be eliminated from further consideration.

Given the parameter ranges and the quantitative relationships of the submodels, minimums and maximums may be determined for each submodel and, subsequently, for each criterion. The ultimate goal of this quantification effort is the production of values which are directly comparable across criteria. A simple means to this end might be simple normalization of the form:

$$X = (x - x_{min})/(x_{max} - x_{min}) \qquad \text{Eq. 2.2}$$

However, it is important to recognize that this relationship is limited by its implication of assumed linearity of importance throughout the range of criterion values. For this and other reasons, subsequent portions of this research will show that use of the mathematics of probability theory is the preferred method of normalization.

Having achieved comparability across criteria, it remains to include consideration of the relative weights in obtaining a single figure of merit for each candidate system. This is typically achieved via the well-known additive weighting method, referred to here as the criterion function:

$$CF_{\alpha} = \Sigma a_i X_i \qquad \text{Eq. 2.3}$$

Given the form of CF_{α}, the resulting values will range between 0 and 1.0.

Analysis of the Parameter (Design) Space

It is possible to conceive of a theoretical hyperspace which has an axis for each identified parameter plus one axis for the resulting value of CF. This has been defined as the "design space" (Ostrofsky, 1977d). The possible parameter values and their relationships to criterion values via the submodels may then be perceived to produce a surface in the hyperspace consisting of the parameter and CF values of all theoretically possible candidate systems. This surface may well contain discontinuities resulting from the functional constraints. Each defined candidate system is represented by a point on the surface and an associated vector of values, one value for each parameter axis. Points on the surface not represented by a defined candidate system may represent alternatives not previously considered or may simply represent unachievable combinations of parameter values.

Analysis of this hyperspace is undertaken to reveal useful information about the sensitivity, compatibility, and stability of the candidate systems as well as indications of system performance. A variety of computer search techniques exist from research in other applications (Box and Draper, 1986; Fletcher and Powell, 1963; Fletcher and Reeves, 1964; Hooke and Jeeves, 1961; Leon, 1966; Taubert, 1968; Shestakov, 1983) which may be used to conduct this

design space search. Sensitivity in this context refers to analysis which searches for those parameters which have the greatest impact on the resulting value of CF. By contrast, compatible parameters are defined as those which have the least impact on CF. Stability analysis refers to the identification of parameter value changes which may produce drastic or unexpected consequences. Recognizing that evaluation of candidate systems at this stage is still based on estimates, unstable parameter combinations may represent risky candidate systems.

Formal Optimization

In this step, optimization of CF is pursued based on information gained in the analysis of the parameter space. First, each candidate system is optimized by selecting the combination of permissible parameter values which maximizes its value of CF. This process is referred to as optimization "within" candidate systems. Next, the optimal candidate system is identified as the candidate system with the highest value of CF. Given the construct of CF, that candidate system will be the one whose CF_{α} is closest to (but not greater than) 1.0. This determination is termed optimization "among" candidate systems. Although there is theoretically only one optimal candidate system, it is quite possible that many candidate systems will have CF values nearly equally close to 1.0. If the degree of accuracy of measurements is less than the magnitude of difference, then for all practical purposes these candidates are equally desirable. In such a

case, other factors may be used to select the preferred alternative or more accurate data may be sought.

As stated previously, it is recognized that the assessment of parameter ranges for each candidate system may have been based on incomplete information. Therefore, it is possible that subsequent design activities may mandate changes in the achievable parameter values for a particular candidate system. Consequent changes in the system's parameter values clearly affect the resulting value of CF. Since the candidate chosen was selected based on its best combination of parameter values, any change in those values will result in a decrease in CF if the original work was accurate. Therefore, when alternative changes are possible, it will probably be preferable to use the results of the analysis of the parameter space to insure changes are made to compatible (less sensitive) parameters rather than sensitive ones. Even so, the resulting degradation of CF_{α} may result in another candidate system defaulting to become the optimal. In this case, theory would indicate that the design process should iterate through all intervening steps using the new optimal candidate system. However, it is recognized that in practice this may be infeasible. Therefore, the new optimal candidate system instead becomes a target for modification efforts or the next design effort.

Projection of System Behavior

The optimal candidate system is analyzed more extensively to examine and predict its ultimate behavior on characteristics of interest. Simulation and/or appropriate analytical techniques may be used for these purposes. As a minimum, the projected performance of the optimal candidate system is examined in the context of each of the four production/consumption phases of the system life cycle. This step also provides the best opportunity encountered so far in the design process to obtain detailed cost estimates for the selected system throughout its life cycle.

Testing and Simplification

While the preceding step provides more detailed information than previously available, those projections are still largely speculative. Therefore, testing is warranted to verify the system's attributes in order to reduce the risks of estimation. Testing may be used to verify hypotheses, generate new information, and expose shortcomings.

As an important by-product of the testing process, recognition is frequently gained of areas in which the design may be simplified or streamlined. Therefore, needless complexities may be removed and simplifications introduced as long as system performance remains comparable or improves.

Detail Activities

Collectively, these steps are uniquely tailored to anticipate and resolve every eventuality of the production/consumption phases by preparing for the implementation and sustained operation of the system. Consequently, the specific activities undertaken are determined by the nature of the system being designed and the environment in which it is to be produced, distributed, operated, and retired. Such activities might include, for example, preparation of technical data (e.g. blue prints, specifications), design of support equipment, plans for distribution, training, operator's manuals, maintenance specifications, replacement parts acquisition requirements, disposition instructions, etc. The detailed activities associated with micro-level design and development approaches would normally fall within this category. In general, however, a plan is developed to accomplish all activities required for each phase in the production-consumption phases.

Summary

The essence of a design methodology closely parallels Simon's (1977) classic stages of management decision making with the appendage of an implementation phase (Hogarth and Makridakis, 1981b): intelligence (the awareness of the problem or need), design (the synthesis of solutions), choice (the selection of a specific candidate system), and implementation (the detail activities). The focus of

this research is the development of a decision making approach (the choice stage) within the context of design methodology. The specific methodology presented in this chapter was selected as a framework for the balance of the research because it has provided a sound framework for previous research (Ostrofsky et al, 1977c, 1978, 1979, 1980, 1981; Folkeson, 1982; Wu, 1983; Peschke, 1986), because it embodies the significant developments in the area of design methodology as described previously, and because it provides the inputs required by the proposed decision process.

The methodology is composed of the three Design/Planning phases of the Feasibility Study, the Preliminary Activities, and the Detail Activities. First developed by Asimow (1962), these phases have been proliferated by other authors and closely parallel alternative groupings. The methodology expressly considers the four system life cycle phases of production, distribution, operations/consumption, and retirement in its identification and formulation of the problem and its projection of system behavior. These descriptors of a system's life cycle appear to be the accepted standard. The methodology takes a decidedly macro-level view of the design effort, utilizes an inductive approach in its synthesis of candidate systems, and requires an expansionist perspective in its modeling and optimization activities.

As a basis for the target decision process, the methodology makes available virtually all of the required inputs. It acknowledges the

complexity of the decision environment both in its synthesis of the largest possible set of candidate systems and in its consideration of multiple, potentially conflicting objectives. Its criterion function pursues a prescriptive approach to decision-making in which optimization is sought. Subjective considerations are formally included both during the determination of relative weights and in the modeling process. The use of relative weights also recognizes the difference in importance of decision criteria. Finally, while the description to this point has not discussed the treatment of criteria interactions, it is at least clear that the methodology permits and recognizes the presence of interaction in the specification of criteria, parameters, and submodels. The subsequent chapters of this research are concerned with the pursuit of a decision making process which integrates <u>all</u> of these various considerations in the best possible manner.

Chapter 3

QUANTITATIVE APPROACHES TO DECISION MAKING

The terms "problem solving" and "decision making" are frequently used interchangeably. However, Smith (1988) has provided the following useful distinction. Problem solving concerns the achievement of a goal or the movement from some unsatisfactory state to a more desirable one. By contrast, decision making implies that a choice must be made among alternatives which have been (or will be) defined. This research is concerned with decision making: the choice among candidate systems.

Eilon (1985) has further differentiated four types of decision making:

1) decision making under deterministic conditions: well defined goals, information fully available, deterministic variables and outcomes,

2) decision making under risk: well defined goals, information fully available, stochastic variables,

3) decision making under uncertainty: goals are well defined but information is incomplete,

4) decision making under ambiguity: goals are unclear,

information is incomplete.

In most cases, we might expect the design environment to at least require decision making under uncertainty since information about the candidate systems is generally estimated and/or projected based on past experience. As Eilon (1985) describes, under uncertain conditions ". . . information about future events is incomplete . . ." and ". . . for a given set of assumptions and performance criteria, a range of possible outcomes can be computed . . ." Even in the best (theoretical) possible case where information about the candidate systems is complete and the designer faces decision making under deterministic conditions, Eilon (1985) observes that "these conditions do not necessarily render problems in this category simplistic or trivial" and that many such problems (e.g. the travelling salesman problem), while highly structured and deterministic, are nevertheless complex and difficult to solve.

The presentation of the design methodology in Chapter 2 included a brief description of one approach (the criterion function) to design decision making. However, a great deal of research concerned with decision making is available which might be useful in pursuit of the stated research goals (Blin, 1977; Bridgeman, 1922; Carlsson, 1983; Cheng and McInnis, 1980; Cohen et al, 1985; Epstein, 1957; Green and Srinivasan, 1978; Harrison, 1975; Holloway, 1979; Ignizio, 1976; Janis and Mann, 1977; Keeney, 1982; Miller, 1970; Roy, 1977; Rubinstein, 1975, 1980, 1986; Sage, 1977; Sakawa and Seo, 1980; Shull, Delbacq, and Cummings, 1970; Simon, 1960, 1978; Tribus, 1969; White and Sage,

1980; Wymore, 1976; Yager, 1981; Zeleney, 1973, 1974, 1976a, 1976b, 1977, 1980, 1981a, 1981b, 1982, 1984). The purpose of this chapter is to review some of the primary existing decision making techniques which are currently used or could be used to facilitate the type of decision making stated as the objective of this research. This review is intended to highlight the strengths and weaknesses of approaches commonly suggested as most promising for this type of decision problem.

Two types of approaches will be reviewed: generic quantitative approaches and approaches which have been pursued specifically for the design decision environment. Each technique will be described both in general terms and in the context of the design methodology framework. Finally, a summary review of the limitations of the existing approaches will be presented to suggest the preferred direction for the remainder of this research.

General Quantitative Approaches

Many solution techniques have been developed since World War II within the field of operations research (Wagner, 1975). Originally conceived to address specific interdisciplinary problems, the most powerful of these approaches have been generalized to permit application to broad classes of problems which lend themselves to common formulations. We will consider three such general approaches:

mathematical programming, decision analysis, and dimensionless analysis. Although there are many forms of mathematical programming, we will confine our discussion to the two most frequently suggested for applications such as those considered in this research: linear programming and goal programming.

Linear Programming

Linear optimization approaches have been described as being among the most commercially successful applications of operations research techniques (Wagner, 1975). In general terms, linear programming (L.P.) attempts to find the mix of decision variable values which maximizes (or minimizes) some objective function or overall measure of value subject to a set of constraints on the values the decision variables may assume. Common applications have included product mix decisions, production scheduling, and transportation or routing problems. Specific solution approaches have been developed for special cases such as transportation and assignment problems as well as for the general case.

The general form of the L.P. (maximization) problem is:

$$\max z = \sum_{j=1}^{n} c_j x_j \qquad \text{Eq. 3.1}$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \quad \text{for } i=1,2,\ldots,m \qquad \text{Eq. 3.2}$$

$$x_j \geq 0 \quad \text{for } j=1,2,\ldots,n \quad \text{Eq. 3.3}$$

In this maximization formulation, z is the objective function value, the x_j are the decision variables, and the c_j are the per unit benefits associated with the x_j. The first series of constraints are sometimes referred to as resource constraints. In each of the m constraints, the a_{ij} are coefficients which form a linear combination of the decision variables (x_j) which is limited by some value b_i. Since the a_{ij} and b_i may assume any value and m may be any positive integer, the constraint set may be of virtually any size or nature in the general form. The special cases mentioned above are applicable when the constraint set assumes specific characteristics. The second series of constraints forces the decision variables to be non-negative: a generally applicable requirement when the variables represent the amount of some physical substance or characteristic.

Note that the formulation of the L.P. problem reflects its inherent applicability and assumptions. The foremost distinguishing characteristic is its assumption of linearity in both the objective function and the constraints. Although this assumption may not be strictly true in many cases, it is frequently close enough to produce reasonable solutions, particularly when weighed against the difficulty of non-linear solution techniques. Closely related to the assumption of linearity is the characteristic of a continuous range of values which the decision variables may assume. Other L.P. formulations and solution techniques are available when integer restrictions apply;

however, the associated computational complexity of these additional restrictions can be substantial.

It is convenient to visualize the L.P. problem and its solution graphically for the case of two decision variables. Assume the example problem:

$$\max z = 7x_1 + 10x_2 \qquad \text{Eq. 3.4}$$

subject to

$$3x_1 + 2x_2 \leq 36 \qquad \text{Eq. 3.5}$$

$$2x_1 + 4x_2 \leq 40 \qquad \text{Eq. 3.6}$$

$$x_1 \leq 10 \qquad \text{Eq. 3.7}$$

$$x_1,\ x_2 \geq 0 \qquad \text{Eq. 3.8}$$

As shown in Figure 12, the constraint set can be seen to define a region of feasible solutions. (An infeasible problem is one in which all constraints cannot be satisfied simultaneously.) Any point within this region identifies a solution which satisfies the constraint set. The objective is then to define that point (or points) which maximizes the value of the objective function. Clearly, this will occur at the boundary of the region of feasible solutions. Graphically, this determination may be facilitated by first plotting a line representing the slope of the objective function which passes through the feasible region. By choosing different values for the objective function, a family of lines may be plotted (Figure 13). Each of these isolines (Dannenbring and Starr, 1981) will be parallel to the others, varying only in its distance from the origin. The optimal solution may be

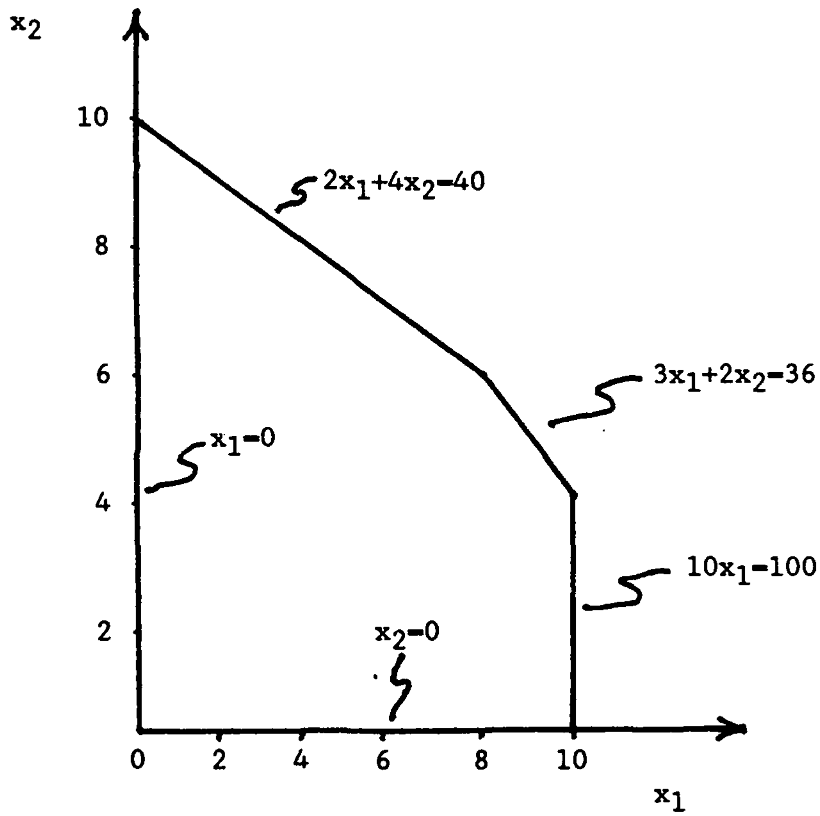


Figure 12. Linear Programming Constraint Set

(Adapted from Dannenbring & Starr, 1981)

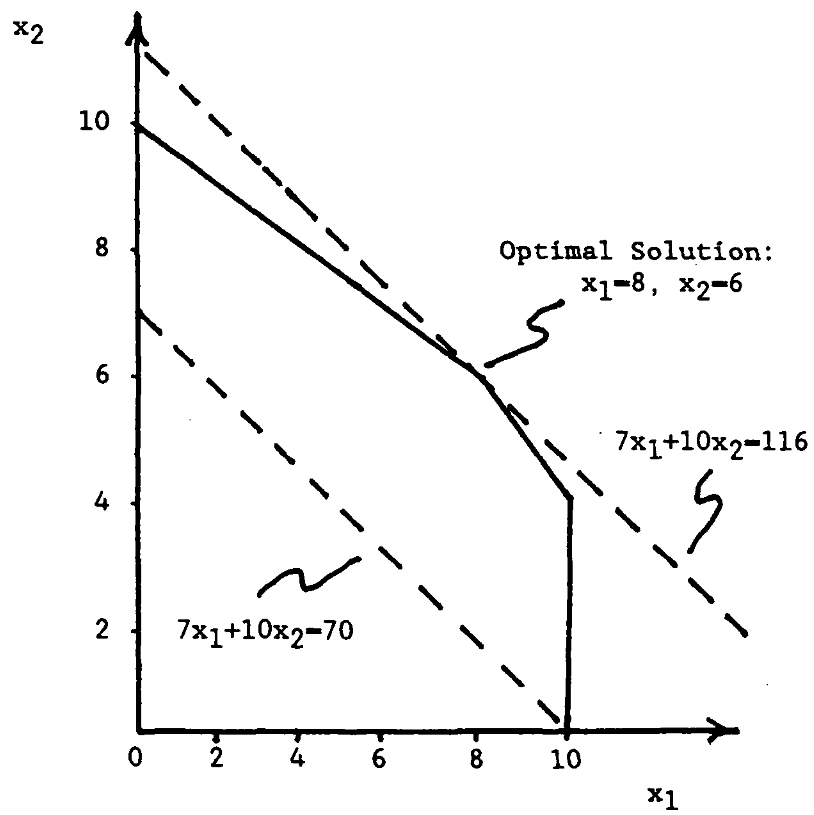


Figure 13. Objective Function Isolines

(Adapted from Dannenbring & Starr, 1981)

obtained by finding the objective function value which produces the isoline farthest from the origin while still touching the feasible region (Figure 13). The values of the decision variables are then determined by the point at which that intersection occurs.

A variety of efficient algorithmic L.P. solution techniques have been developed. Perhaps the best known of these is the simplex, devised by Dantzig (1963). Other efficient methods utilize the dual formulation of the L.P. problem. We will not describe these techniques since the method of their solution is not relevant to the direction of this research. What _is_ relevant is that these methods produce provably optimal solutions subject to the limitations of data accuracy and linearity of the objective function and constraining equations.

The L.P. approach might be applied to the design decision problem in at least two ways depending on how we define the decision variables. The decision we ultimately desire is the determination of the optimal candidate system. Since a candidate system is distinguished by its set of parameter values $\{y_k\}$ it may be wise to use the parameters as decision variables. Following this approach, the regional and functional constraints would define the feasible region. (Note, however, that the functional constraints may produce discontinuities.) Using constant relative weights, the objective function would then effectively consist of the criterion function with all the submodels embedded. Notationally, we would have:

$$\max CF = \Sigma a_i x_i \qquad \text{Eq. 3.9}$$

subject to

$$\sum_k c_{kl} y_k \leq d_l \quad \text{for all } l \qquad \text{Eq. 3.10}$$

where

$$x_i = f_i\{z_j\}, \text{ and} \qquad \text{Eq. 3.11}$$

$$z_j = g_j\{y_k\} \qquad \text{Eq. 3.12}$$

(An equivalent alternative would be to define the submodels as constraints.)

Unfortunately, this formulation would require that all constraints and submodels be represented as linear combinations: a restriction not inherent in the design methodology itself and seldom true in complex systems. In addition, the resulting solution would specify some combination of parameter values which may or may not represent a realizable candidate system. It might be possible in simple cases to add constraints to increase the likelihood of realizability. However, the ability to successfully define such constraints as a general rule is speculative and the additional solution complexity could easily become prohibitive.

An alternative formulation could be conceived in which the criterion values serve directly as decision variables:

$$\max CF = \Sigma a_i x_i \qquad \text{Eq. 3.13}$$

subject to

$$\sum_i c_{il} x_i \leq d_l \quad \text{for all } l \qquad \text{Eq. 3.14}$$

In this formulation, all the functional and regional constraints would have to be directly translated to linear combinations of the $\{x_i\}$. This translation could well be impossible. Even in the case where such a translation is possible, the optimal values of the decision variables $\{x_i\}$ would still have to be "un-translated" back into some combination of parameters $\{y_k\}$. This too might prove infeasible or difficult at best. Finally, even if such a conversion is achieved, the combination of $\{y_k\}$ may not represent a realizable candidate system.

Goal Programming

L.P. is generally considered to be a single objective approach since it seeks optimization of a single objective function. Even though our design decision problem is inherently multi-objective, we were able to consider application of L.P. by using constant relative weights $\{A_i\}$ to construct a single linear combination of the multiple decision criteria. An unfortunate consequence of this simultaneous consideration was the computational complexity introduced by the interaction of submodels due to shared parameters. An alternative to L.P. which was devised for the express purpose of considering multiple objectives is goal programming (Charnes and Cooper, 1960; Ignizio, 1976; Lee, 1972).

An extension of the basic approach to L.P., goal programming

recognizes and attempts to satisfy a rank-ordered set of goals or objectives. If it is unable to simultaneously achieve all objectives, it attempts to minimize the undesirable deviation from objectives, insuring higher order objectives are satisfied first (Dannenbring and Starr, 1981). It may also be formulated and solved without respect to any rank ordering of goals, striving instead to minimize the total simultaneous deviation from all goals (Hillier and Lieberman, 1980).

The problem formulation is similar to that of L.P. Technological constraints are formed as before to deal with resource limitations and other restrictions. In addition, each goal is converted to a goal constraint (Dannenbring and Starr, 1981) which includes a slack or surplus variable which represents deviation from the desired level of goal achievement. The objective is then stated as the minimization of the (undesired) deviations in the goal constraints. In the case where goals are rank ordered, the objective would be of the general form:

$$\text{Minimize } D_j^- \mid \text{Minimum } D_i^+ \qquad \text{Eq. 3.15}$$

and is read as, "minimize D_j^- given the minimum D_i^+", where D_j^- and D_i^+ are undesirable deviations from the jth and ith goals and where goal i has higher priority than goal j (Dannenbring and Starr, 1981).

In the case where all goals are considered of equal importance (priority), the objective would be stated as:

$$\text{Maximize } z = \min(z_1, z_2, \ldots, z_k) \qquad \text{Eq. 3.16}$$

which is read as, "maximize the minimum progress toward all objectives" (Hillier and Lieberman, 1980). Differing in form from the first case, this problem would be formulated as:

$$\max Z = z \qquad \text{Eq. 3.17}$$

subject to

$$\sum_{j=1}^{n} c_{jk} x_j - z \geq 0 \quad \text{for } k = 1,2,\ldots,K \qquad \text{Eq. 3.18}$$

$$x_j \geq 0 \quad \text{for } j = 1,2,\ldots,n \qquad \text{Eq. 3.19}$$

In the latter case, solution is accomplished using standard L.P. solution techniques. In the former, solution may be achieved with only slight modification of the simplex method (Dannenbring and Starr, 1981).

With respect to the design decision environment, goal programming provides an alternative with respect to the treatment of multiple objectives. Specifically, it permits optimization of design criteria in an ordinal manner, eliminating the necessity for assigning relative weights. In cases where determination of relative weights is an insurmountable problem, this is a clear advantage.

However, as is the case with L.P., goal programming does not provide a means for treating varying relative weights. Furthermore, a price is incurred with its use. The tradeoff is that a coarser distinction exists among criteria since each higher priority is

completely satisfied (to the extent possible) before the next lower priority criterion is considered at all.

Another characteristic common to L.P. is that if interaction is expected or observed among decision criteria, the interaction would likely be included in the model as an additional constraint. Unfortunately, by adding such a constraint, the decision maker is essentially restricting the design space in order to mandate interaction rather that simply responding to its presence. Therefore, superior solutions which might deviate from the "normal" pattern of interaction will not be considered. Furthermore, since the interaction is modeled as a constraint rather than being included in the objective function, the decision maker is unable to express the relative importance attached to joint criteria effects in the selection process.

Additionally, the method requires specification of target levels of achievement which have a direct bearing on the degree to which objectives may be achieved. In the uncertain environment of design, these targets would be highly speculative and might easily serve to screen possible solutions whose ability to achieve substantial improvements in lower level objectives would more than offset a small improvement on a higher level objective. The specification of such targets can reasonably be described as encouraging satisficing rather than optimizing behavior (Dannenbring and Starr, 1981).

Finally, goal programming makes no improvement over L.P. in reducing the complexity of relating criteria values to specific combinations of parameter values. Neither does it offer any additional guarantees that the solution method obtained will represent a realizable candidate system. While it is generally true that data characteristics may be obtained from a real (or potentially real) system, it is not necessarily true that a real system may be achieved from any set of data values. In that sense it is fair to say that neither L.P. nor G.P. directly evaluate specific candidate systems. Instead, they seek to identify the theoretically best data values that could be obtained under the stated conditions.

Decision Analysis

Decision analysis, also referred to as utility theory, has evolved as an alternative approach to multi-criteria decision making which strives to closely mirror the tradeoff preferences of the decision maker while retaining a prescriptive perspective. Stemming from early work on game theory by von Neumann and Morgenstern (1944), the technique has produced a large body of literature (Bell, Keeney, and Raiffa, 1977; Cochrane and Zeleney, 1973; Farquhar, 1977, 1980; Fishburn, 1964, 1970, 1972b, 1989; Greenwood, 1969; Howard, 1980, 1988; Huber, 1974a; Keeney, 1978, 1982; LaSalle, 1978; Lifson, 1972; MacCrimmon, 1973; Raiffa, 1968; Starr and Zeleney, 1977) which combines quantitative rigor with a distinct goal of practical application on large and complex problems. Recent noteworthy

developments include Saaty's (1977, 1980, 1982, 1988) Analytic Hierarchy Process approach to structuring objectives and Harvey's (1986, 1988) approach to the consideration of long term consequences.

The paradigm of decision analysis (Keeney and Raiffa, 1976b) is shown in Table 5 and is described as follows. In the Preanalysis step, a unitary decision maker confronts a defined problem with a specified set of alternative actions. In the Structural Analysis step, the decision maker constructs a decision tree such as the simple example in Figure 14. The tree is composed of the alternative actions available to the decision maker and the possible outcomes or consequences associated with each action. During Uncertainty Analysis, the decision maker assigns a probability to each possible outcome. (Note here that Keeney and Raiffa, like most decision analyst researchers, use the term "uncertainty" somewhat differently from the way it was used by Eilon (1985). Whereas Eilon used "uncertainty" to simply indicate incomplete information, Keeney and Raiffa use the term to indicate the possibility of many different outcomes, each with some probability of occurrence. This definition corresponds most closely to Eilon's "decision making under risk." For the remainder of this section, the term "uncertainty" is used in the sense intended by Keeney and Raiffa.) Next, the decision maker undertakes a Utility or Value Analysis. This consists of indicating his or her preferences by assigning a utility value to each possible consequence. In the case of uncertainty (i.e. multiple consequences for each action, each with an associated probability), the expected

TABLE 5.

PARADIGM OF DECISION ANALYSIS

Preanalysis

Structural Analysis

Uncertainty Analysis

Utility or Value Analysis

Optimization Analysis

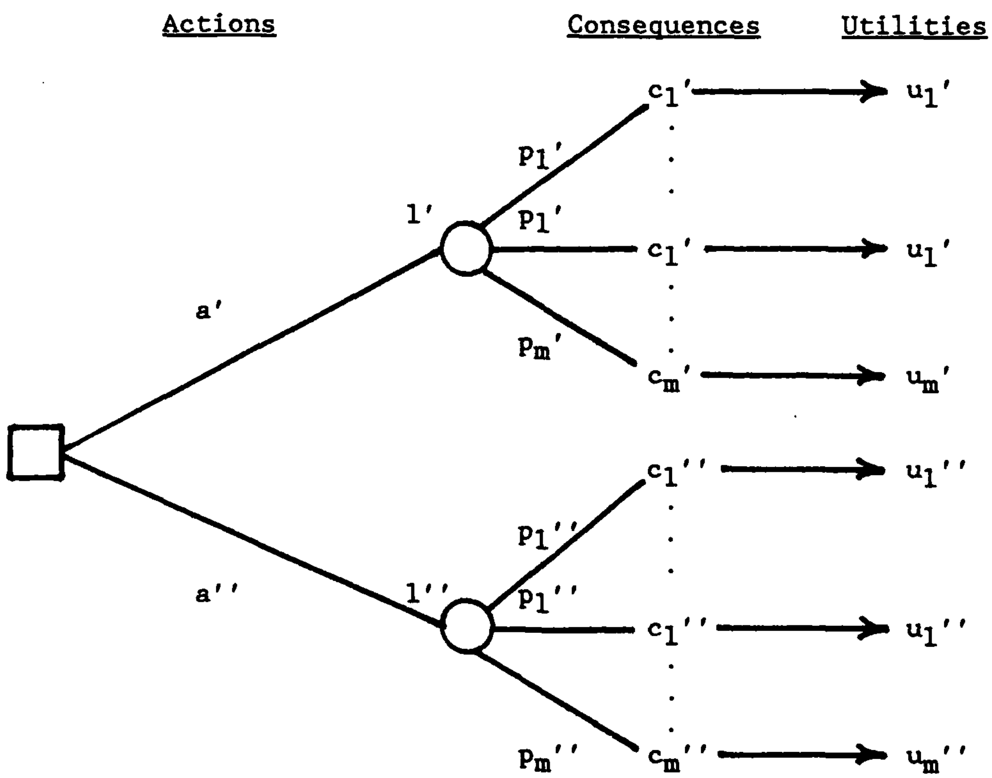


Figure 14. Decision Tree

(Keeney & Raiffa, 1976b)

utility of any action is computed as:

$$\sum_{i=1}^{m} p_i u_i \qquad \text{Eq. 3.20}$$

where p_i and u_i are the probability and the utility associated with the ith consequence. Finally, in Optimization Analysis, the decision maker selects that strategy of actions which maximizes expected utility. In cases where a series of actions is required, dynamic programming is suggested as a means of determining the optimal strategy.

When consequences can be described in terms of a single attribute (criterion), the determination of the optimal decision is a relatively straightforward matter as described above. However, in the case of multi-attribute decisions, some means of assessing joint preference over all attributes simultaneously must be found. This desired value function is denoted as:

$$v(x_1, x_2, \ldots, x_n) = f[v_1(x_1), v_2(x_2), \ldots, v_n(x_n)] \qquad \text{Eq. 3.21}$$

where v_i represents a value function over the single attribute X_i. The bulk of decision analysis is concerned with the search for and/or the verification of this function which is said to define the decision maker's preference structure.

At this point, it is appropriate to distinguish the terms value function and utility function. Value functions apply when consequences are assumed to be known with certainty, i.e. the deterministic case (Keeney and Raiffa, 1976b: 68). When, as described earlier, uncertainty prevails so that consequences have associated probabilities, i.e. the stochastic case, the applicable measure of preference is termed a utility function (Keeney and Raiffa, 1976b: 16). Although an extension of this work could be made to the stochastic case, we have so far in the design context not defined uncertain states of nature or their associated probabilities. Therefore, unless otherwise specified, we will confine our discussion to the use of value functions.

One decision alternative is said to dominate another when the first alternative is preferred to the second with respect to each and every attribute considered independently. Visualized graphically in the simple case of two dimensions (Figure 15), we may plot a region of possible consequences (related to possible action alternatives) as points represented by their raw values for each of the two attributes. All points (consequences) not dominated by any other point are collectively referred to as the efficient frontier or Pareto optimal set (Keeney and Raiffa, 1976). Clearly, only points on the efficient frontier should be considered for implementation and the optimal strategy must lie on the efficient frontier.

While the efficient frontier addresses the relative performance

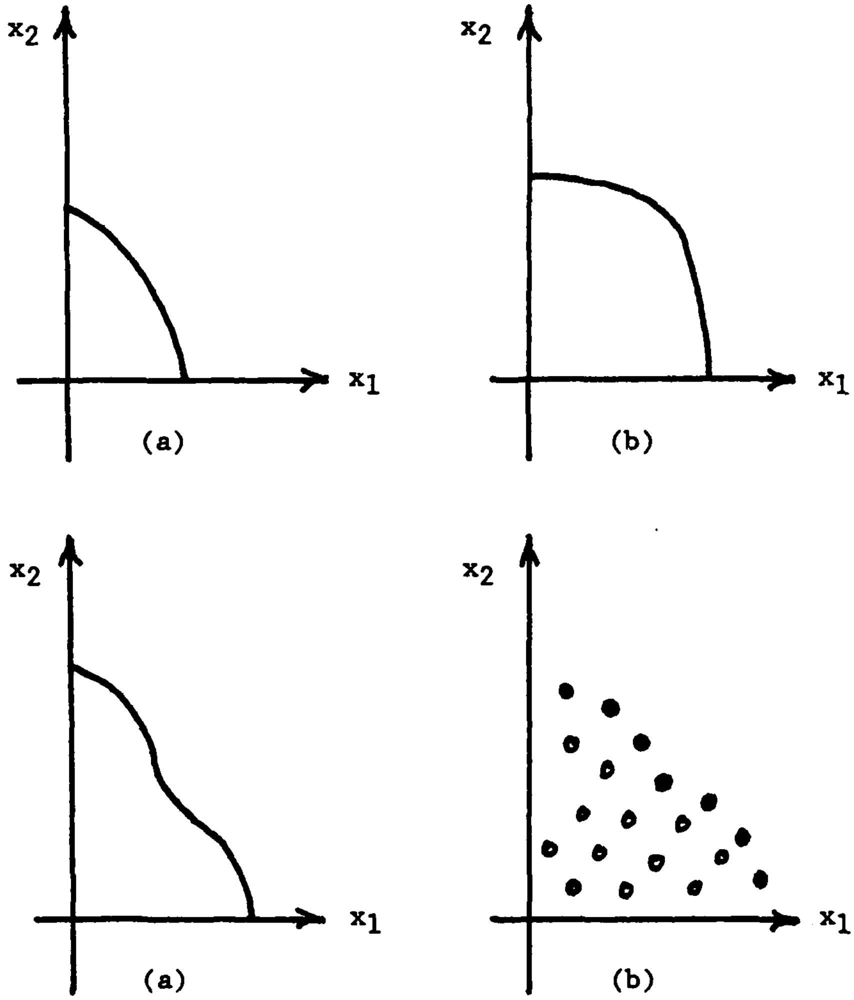


Figure 15. Efficient Frontier of Decision Alternatives

(Keeney & Raiffa, 1976b)

of the decision alternatives, it provides no information about the preferences of the decision maker. Toward this end, points in the decision space considered of equal preference may be connected to form what are referred to as indifference curves (Figure 16). Any two points on the same curve are equally preferable. A set of preference curves is collectively described as a preference structure. It can then be seen that the optimal choice is the point farthest from the origin at which the efficient frontier intersects the decision maker's preference structure (Figure 16). Restated, the optimal choice is that alternative which has the greatest preference value associated with it. Consequently, the decision analyst endeavors, through interaction with the decision maker, to establish the value functions which define the preference structure. An alternative to the mapping of the complete preference surface is the hierarchical structuring of objectives and alternatives to permit the specification of preferences in ratio form via pairwise comparisons (Saaty, 1977, 1980, 1982, 1988; Olenik and Haimes, 1979).

The concept of independence is crucially important in the definition of value functions. Attribute X_1 is said to be preferentially independent of X_2 if the preference structure in the X_1 space does not depend on the value of X_2. The presence of mutual preferential independence among the decision criteria means that the decision maker's preference structure may be specified by determining the value functions for each of decision attribute considered independently of the other attributes. Under this condition, an

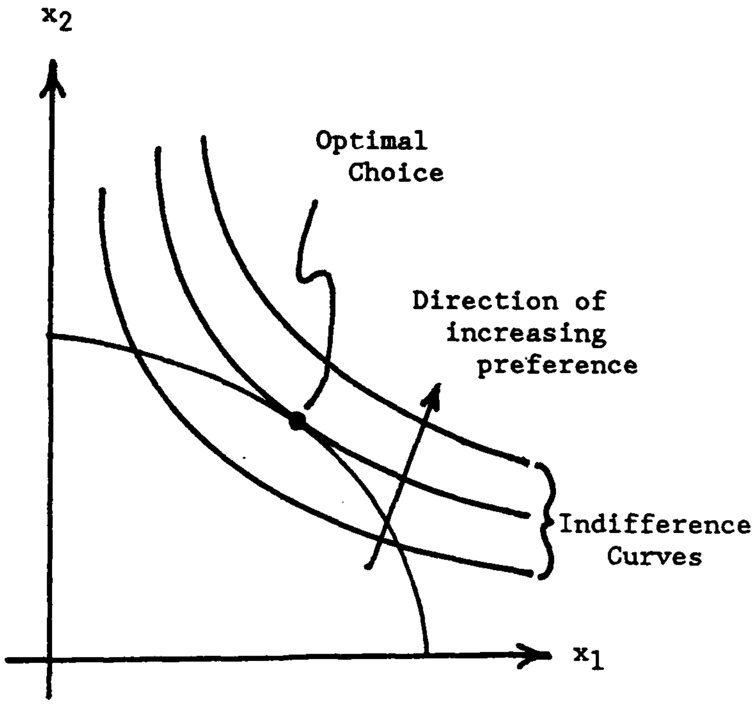


Figure 16. Indifference Curves

(Keeney & Raiffa, 1976b)

additive value function is defined as

$$v(x_1,x_2,\ldots x_n) = \Sigma v_i(x_i) \qquad \text{Eq. 3.22}$$

If the preference structure of X_1 does depend on the value of X_2, then interaction is present and the determination of the preference structure becomes far more complex. In this case, extensive additional discussion must ensue between decision maker and decision analyst. Rather than being able to simply define marginal (single-value) attribute preferences, it may be necessary to attempt to define the complete multi-dimensional preference structure surface by painstaking comparison of individual alternatives. Not surprisingly, much effort has been devoted to the achievement and verification of the preference function or of preferential independence (Bell, 1979; Briskin, 1966, 1973; Dewispelare and Sage, 1979; Dyer and Sarin, 1979; Fishburn, 1972a; Keeney, 1972, 1973a, 1976a; Kirkwood and Sarin, 1980; Taylor, 1973). Significant additional research has been devoted to practical issues such as sensitivity analysis (Barron and Schmidt, 1988; Evans, 1984) and applications (Bard and Feinberg, 1989; Crawford, Huntzinger, and Kirkwood, 1978; de Neufville and Keeney, 1974; Fishburn, 1967; Huber, 1974a; Keeney, 1973b, 1988; Mehrez and Sinuany-Stern, 1983).

In the context of the design methodology, the value or utility associated with each attribute for a candidate system is most accurately described as the product a_iX_i since it simultaneously

considers both the quantity of the attribute possessed and the decision maker's relative weight. Consequently, it becomes difficult to differentiate whether the specified value of an alternative is predominantly influenced by the objective performance of the candidate system or the decision maker's preference. For example, take the case of two candidate systems and their performance with respect to one attribute. Assume candidate system #1 has an observed value of 333 on the attribute of interest and candidate system #2 has a value of 344. Further assume that the decision maker's value function assigns values of .2 and .8 respectively to the two candidates on a scale of 0 to 1.0. Is the magnitude of the difference between the two values primarily an indication that 344 is a substantially greater attribute score than 333 in an absolute (or objective) sense or is it simply a reflection of a rapid shift in the relative importance of the attribute as perceived by the decision maker? Certainly it could be argued that the end effect is the same regardless of the reasons which led to it. However, this failure to separately consider the objective performance (X_i) and the subjective input (a_i) contributes to the difficulty in achieving a consistent and meaningful preference structure. Clearly, the difficulty resulting from failing to distinguish these two factors is compounded in the case where it is necessary to simultaneously consider multiple interacting attributes (criteria).

Dimensionless Analysis

The final general quantitative approach we review is that of dimensionless (or dimensional) analysis (Bridgeman, 1922). Dimensionless analysis is a simple, essentially heuristic technique which is expressly concerned with comparing alternatives in terms of multiple decision criteria expressed in noncomparable units.

Given a pair of alternatives, the technique eliminates criteria units by forming the ratio of alternative A's performance (in raw units) to alternative B's on each criterion. The only requirement for criterion measurements is that the direction of preference is common across criteria, i.e. high raw criterion scores are always good or always bad. If we assume that low scores are preferred, then alternative A is preferable to B for a given criterion if the ratio of their raw scores is less than 1.0. If the ratio is greater than 1.0, B is preferred.

Each criterion is assigned a relative weight on the scale 1 to 10 and the performance ration for each criterion is raised to the power of the relative weight. The resulting weighted ratios are then multiplied across all criteria to produce a single number. Notationally, this may be shown as

$$\prod_{i=1}^{n} (A_i/B_i)^{W_i} \qquad \text{Eq. 3.23}$$

where A_i and B_i are the raw performance scores for alternatives A and B on the ith criterion and where W_i is the relative weight of that criterion. As before, if the overall result is less (greater) than 1.0, A (B) is preferred. Clearly, the higher the relative weight of a given criterion, the greater the role that performance ratio plays in the overall decision. An example is shown in Table 6.

While the method explicitly considers only a pair of alternatives at a time, it might perhaps be extended to consider multiple alternatives. Alternately, a simple tournament-style pairing approach can be used in which the winner of one pairing can be "played off" against the winner of another pair.

Easy to use and logical, the method must nonetheless be considered heuristic since the results will be affected by the initial scale of raw criterion measurements. For example, if temperature was a criterion, the performance ratio and, therefore, the results (although not necessarily the ultimate decision) would be affected by whether temperature was measured in Celsius or Fahrenheit. In addition, the method lacks an established theoretical basis. Therefore, it is debatable whether independence among criteria is implicitly assumed.

TABLE 6.

DIMENSIONLESS ANALYSIS

Criteria	Raw Scores A	Raw Scores B	Ratio	Weight	Weighted Ratio
Weight	37	39	.949	1	.9490
Cost	6725	4550	1.480	3	3.2400
Volume	113	142	.796	1	.7960
Safety (inverted)	2	5	.400	4	.0256
Speed (inverted)	175	75	2.330	2	5.4400
RESULT					0.3410

(A is preferred)

Design-Based Approaches

The quantitative approaches just discussed are general purpose in nature and have proven their worth in innumerable applications. However, it is frequently the case that a general purpose tool, while useful in a variety of scenarios, may not be as useful in any specific case as a tool developed especially for that purpose. For this reason, decision making techniques have been developed expressly for use in the macro-level system design context. In the following paragraphs we review two such approaches: the Tricotyledon theory of system design and the criterion function.

Tricotyledon Theory of System Design

Developed by Wymore (1976), this approach to the design decision is deeply rooted in systems theory. It pursues the use of set theory and mathematics as a common basis for combining theories from a variety of disciplines within the context of design (Wymore, 1967).

The approach begins with a problem statement in the form of a target system definition. Based on the system definition, an input/output specification is developed which attempts to define all possible combinations of inputs over time (input trajectories). Similarly, the plan defines all possible output trajectories and attempts to match each input to a possible set of outputs. Next, a set of systems is defined, each of which satisfies the input/output

specification over some subset of time. In other words, for any defined input, the system will produce one of the matched outputs for some period of time. This set of systems is referred to as the input/output cotyledon.

A separate examination of technology is then conducted in order to identify all systems that can be assumed to be available as components within the time span of the design process. These components are then combined via coupling recipes to form potential systems. The resulting set of systems is termed the technology cotyledon.

The feasibility cotyledon is then defined as the intersection of the input/output and technology cotyledons (Figure 17). Each system in the feasibility cotyledon satisfies the input/output specification over some period of time and can be achieved from a coupling recipe using components from the technology. This set is effectively the set of candidate systems.

A merit ordering process is then used to select the optimal candidate system. The process suggested by Wymore (1976) for this purpose is essentially hierarchical (Figure 18). First, experiments are constructed to measure characteristics of interest. For systems in the input/output cotyledon, these characteristics are typically benefits, while the characteristics of interest in the technology cotyledon are usually costs. A performance index is assigned as the

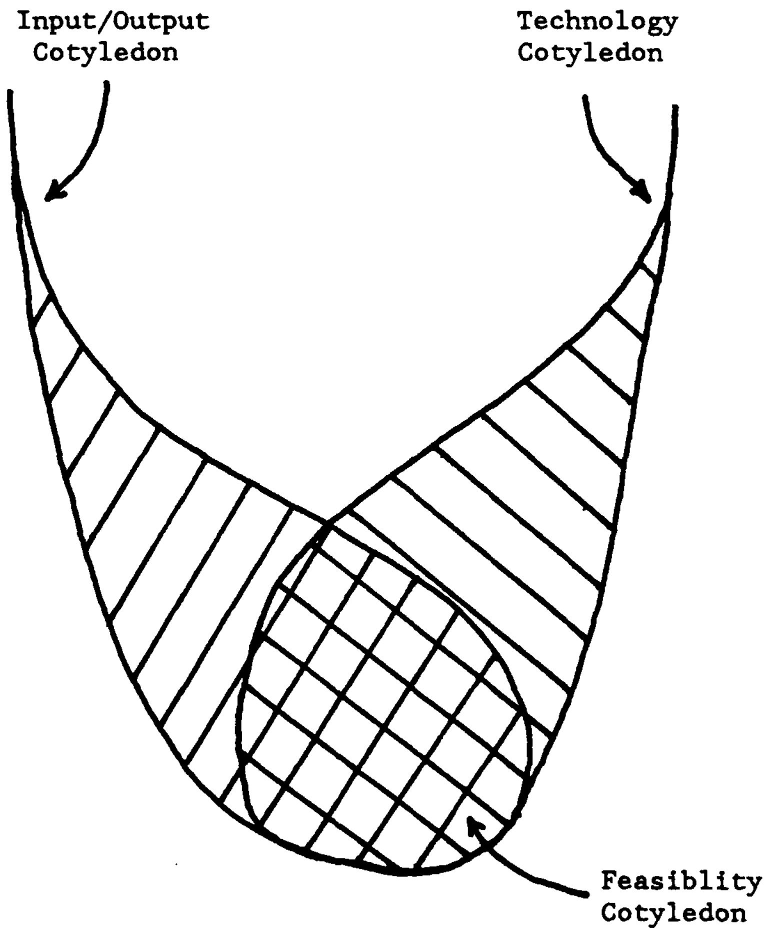


Figure 17. Tricotyledon Theory of System Design

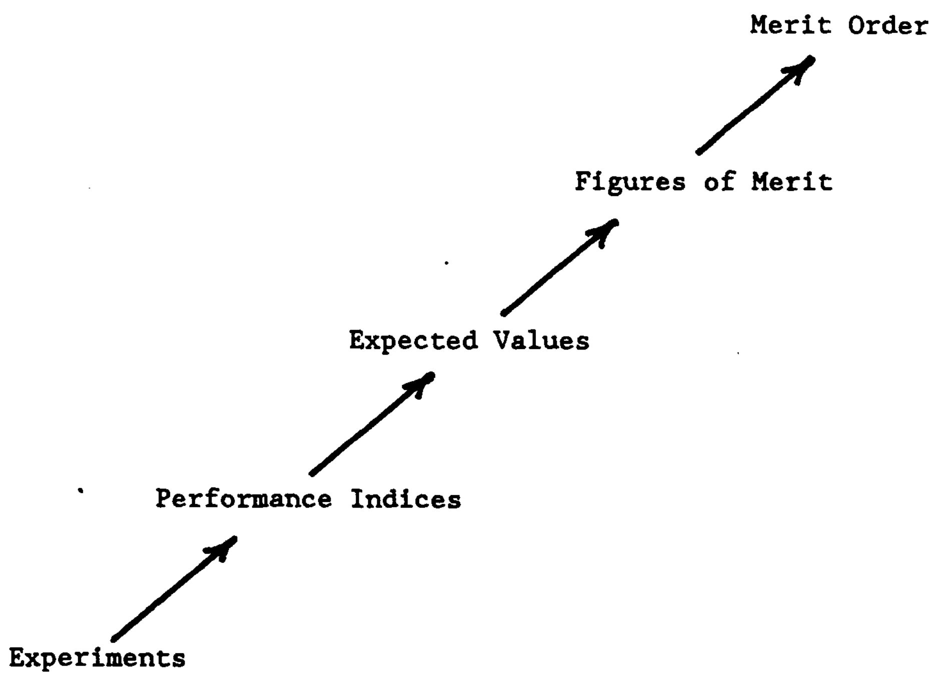


Figure 18. Merit Ordering

raw performance score of a system on one experiment. Expected values are then obtained by weighting and combining the performance indices for all experiments on a system. Figures of merit consist of expected values of performance indices. Finally, an order of merit is obtained by weighting and combining figures of merit. This merit ordering process is first conducted independently over the input/output and technology cotyledons. A final merit order is defined over the feasibility cotyledon in such a way that it is consistent with the first two merit orderings while specifying tradeoffs between the two to permit an overall ranking of candidate systems.

If implemented rigorously, the Tricotyledon approach can clearly be a resource-intensive process. While its broad systems perspective increases the chances of considering the theoretic optimum, the magnitude of defining all possible inputs and outputs over time represents an impressive task of thoroughness. In addition, the difficulty of identifying the intersection between the input/output and technology cotyledons cannot be overemphasized.

Subjectivity and the inclusion of relative weights are permitted to enter the merit ordering process in a variety of ways. Therefore, the potential exists for inconsistency or redundancy in the weighting process. However, the flexibility of the approach permits the exact structure of the quantitative evaluation process to be tailored to the application. The effectiveness of the process is therefore difficult to generalize, depending largely on the skills of the designer.

Criterion Function

Resulting from original work by Asimow (1962) and extensions by Ostrofsky (1968, 1977d) the criterion function approach to the design decision problem was described in general terms in Chapter 2. In the remaining paragraphs of this chapter we provide a more detailed description comparable to that presented for the other decision techniques.

As described earlier, design criteria are generally not specific enough be directly measurable. Consequently, each criterion is logically broken down into constituent elements. Each element, in turn, is likewise analyzed until a set of measurable characteristics (parameters) is identified which is sufficient to represent the criterion. Those criterion elements which serve as intermediate links between criteria and parameters are referred to as submodels. Quantitatively, then,

$$z_j = g_j(y_k) \qquad \text{Eq. 3.24}$$

$$x_i = f_i(z_j) \qquad \text{Eq. 3.25}$$

$$\text{and} \quad x_i = f_i(g_j(y_k)) \qquad \text{Eq. 3.26}$$

where y_k represents the kth parameter, z_j represents the jth submodel, and x_i represents the ith criterion.

The ultimate combination of the various criteria measures into a single figure of merit requires conversion to unitless, consistently scaled values. The conversion method described earlier is simple linear normalization of the form:

$$X = (x - x_{min})/(x_{max} - x_{min}) \qquad \text{Eq. 3.27}$$

However, to avoid the assumption of linearity of importance and to provide a means for the treatment of criteria interaction, the preferred method is the use of probability theory. Specifically, the raw criterion scores of all candidate systems are combined to form a cumulative distribution function (c.d.f.) for each criterion (Figure 19). Each value of x_i is then converted to some value of X_i on the range 0 to 1 using the appropriate c.d.f.

It may be useful at this point to pause and consider the use of probability theory in this line of research. To many readers, the term "probability" invokes images of random events, repeated observation of outcomes, and the quantitative description of the relationships between them. However, as Savage (1954), Anscombe and Aumann (1963), and many others have pointed out, probability has equally important meaning and applicability when used in a much broader, logical, subjective sense. Concerned primarily with personal preferences, as opposed to the strict concept of chance, this latter meaning serves as the philosophical and quantitative basis for the application of probability in both utility theory and the criterion

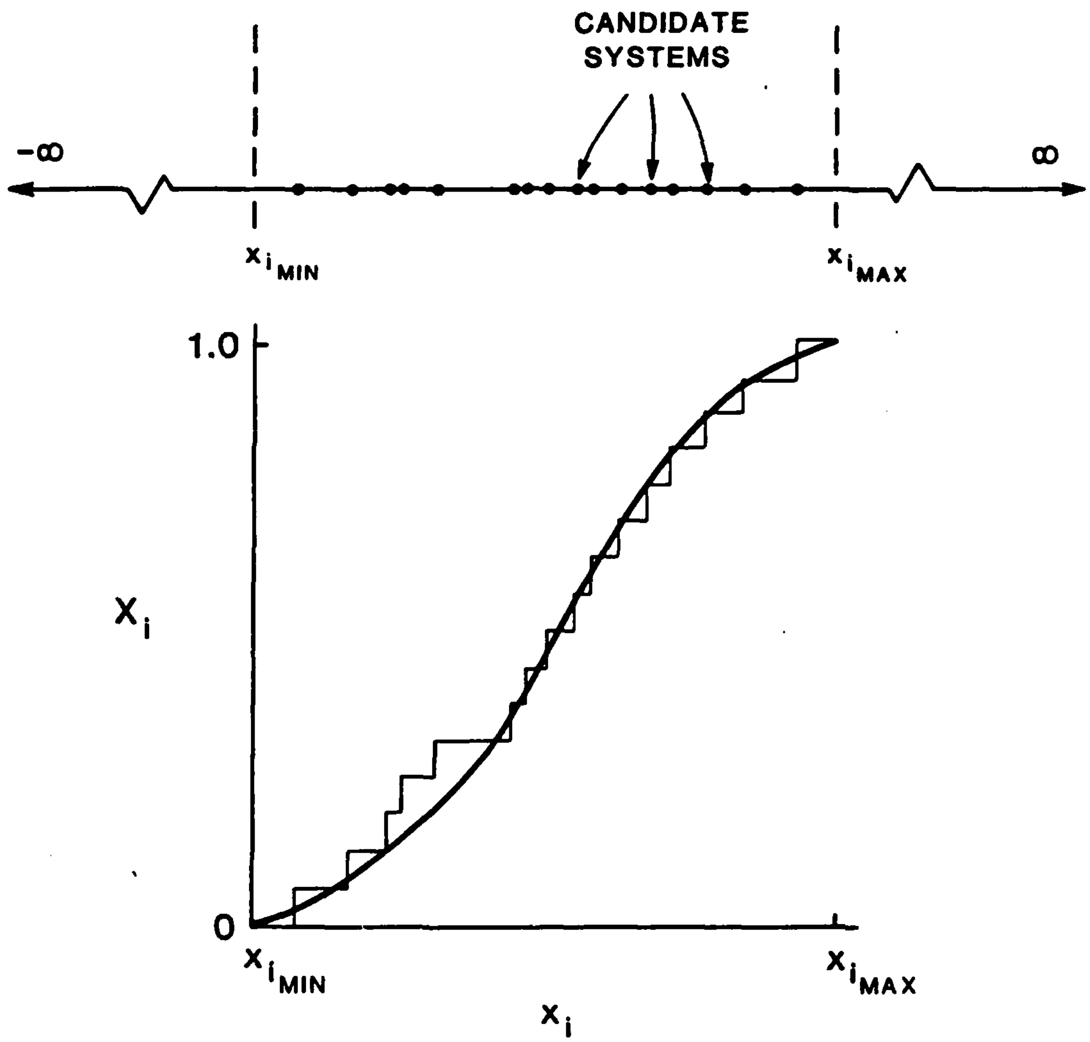


Figure 19. Criterion Conversion Using

Cumulative Distribution Function

(Simons & Ostrofsky, 1988)

function approach described in this paper. It is imperative to remember that our objective is to use the theory as an objective tool to help insure an optimal (Keen, 1977) choice can be achieved, given the inherently subjective nature of the environment.

To this point, the discussion has focused exclusively on the observable or objective characteristics of the system being planned. However, in most cases of complex systems, it is unlikely that all design criteria are considered to be of equal importance. Although it may be reasonable to include such subjective considerations, it is desirable to treat these considerations as objectively as possible. Consequently, each criterion is assigned a relative weight (a_i). Although any of a number of methods may be used to obtain these relative weights (Huber, 1974b; Choo and Wedley, 1985; Eckenrode, 1965; Gershon, 1984; Nutt, 1980), they are normalized in such a way that:

$$\Sigma a_i = 1.0 \qquad \text{Eq. 3.28}$$

and probability theory may, again, be used as a guide in subsequent manipulations.

In many cases, it is unreasonable to expect that relative weights will remain the same for all possible criterion values. For example, consider the simplistic case where only two criteria, cost (x_1) and operational availability (x_2), are being considered. Assume that a

project budget has been established at the midpoint of the range of project costs considered plausible. Up to the point where the budget is exceeded, maximizing availability may be far more important than the money required to achieve it. Once the budget is exceeded, however, the difficulty of obtaining additional funds suddenly becomes substantially more difficult. Beyond that point, the cost criterion may be considered far more important (relatively) than the higher availability obtained. This situation is illustrated in Figure 20 where higher values of X_i indicate <u>better</u> performance on each criterion. (Therefore, the budget is exceeded at values of X_i <u>below</u> the mid-point.)

The first four classes of models (I-IV) in Figure 21 have been differentiated based on the behavior of the relative weights over the range of their respective criteria. In Model I, the relative weights for all criteria are constants. In Model II, the relative weights of all criteria are constant within specified intervals; however, the relative weight of at least two criteria change at least once throughout the X_i range. Model III may be thought of as an extension of Model II to the case where an infinite number of intervals exist such that the relative weights are approximated by continuous functions which vary throughout their ranges. Model IV may be considered a combination of Models II and III.

In order to obtain a single figure of merit for each candidate system, the criteria values and their respective relative weights are

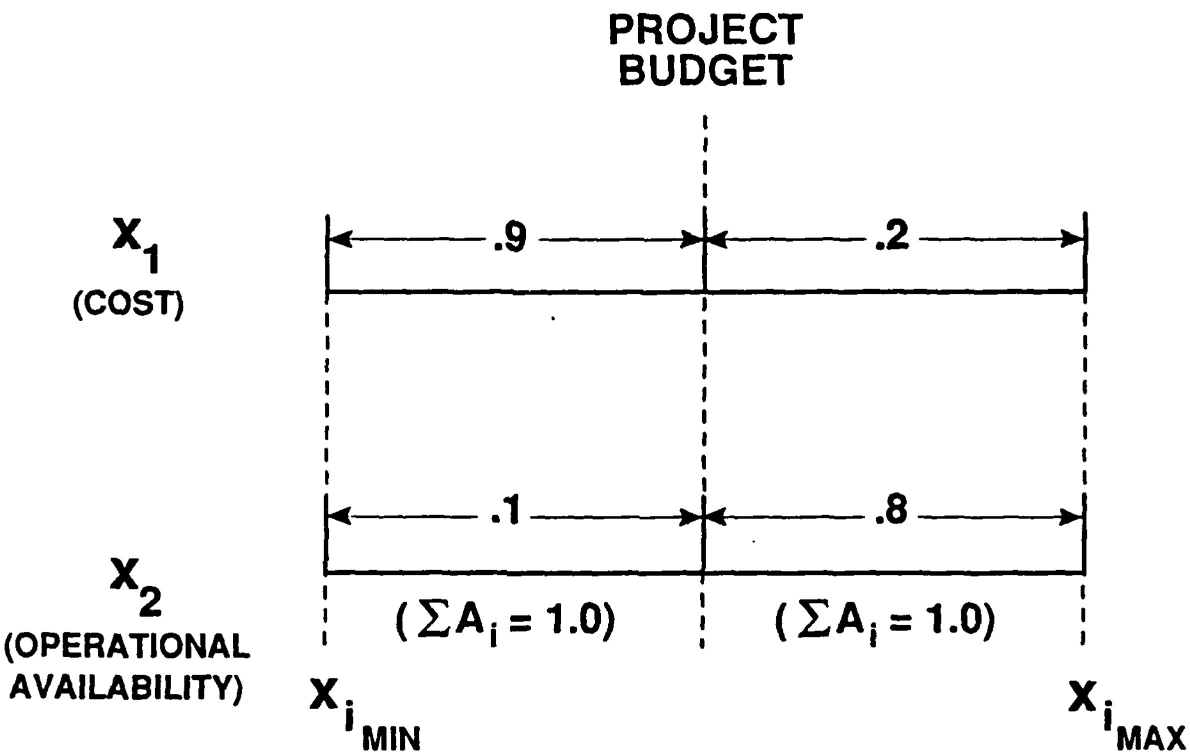


Figure 20. Varying Relative Weights

(Simons & Ostrofsky, 1988)

CHARACTERISTICS OF CRITERIA

NATURE OF RELATIVE WEIGHT

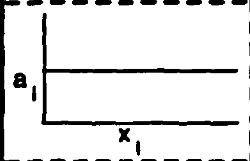
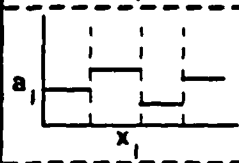
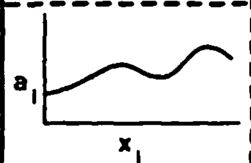
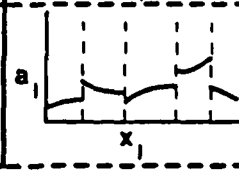
	INDEPENDENT	WITH INTERACTION	GRAPHIC REPRESENTATION OF RELATIVE WEIGHTS
CONSTANT	I	V	
INTERVAL	II	VI	
VARIABLE	III	VII	
VARIABLE WITH DISCONTINUITIES	IV	VIII	

Figure 21. Eight Classes of Criterion Function Models

(Peschke, 1986)

combined in the form of a criterion function (CF). The form most frequently used for this purpose has been the additive form, which sums all criterion values, with each multiplied by its relative weight:

$$CF = \Sigma(a_i)(X_i) \qquad \text{Eq. 3.29}$$

The value of CF is computed for each candidate system. The candidate with the highest value of CF is considered the optimal candidate system. While the theoretical optimal system would have CF = 1.0, there is no assurance that such a value will be obtained since the theoretical optimal combination of parameter values may not represent a realizable candidate system. (In applications to date, no such candidate system has emerged.) However, a search of the design hyperspace for such a theoretical candidate is still useful, since a better candidate may have been overlooked. Even if not, the location of higher values of CF may represent desirable directions for future improvement.

Unfortunately, the strictly additive form of the criterion function may not always be appropriate. As mentioned earlier, multiple criteria frequently have interactions among them. Clearly, our previous example was based on a direct interaction between cost and availability. It is largely for the purpose of dealing with such interactions that probability was chosen as the preferred method for generating normalized X_i.

The first step in determining criteria interaction (X_{ij}) is to verify its existence. This may be done visually by plotting the values of each candidate system on a graph whose axes correspond to the interacting criteria. The resulting pattern (the functional interaction) may then be approximated as a continuous function (e.g. via regression) as illustrated in Figure 22. Statistical validity of the approximated function may be supported using a variety of nonparametric tests (e.g. Kolmogorov-Smirnov). Based on current practice, this function ($f(x_i,x_j)$) is then converted to a c.d.f. of the form: $F(x_i,x_j|x_j)$. Using Bayes's Theorem, the interaction term (X_{ij}) may then be computed by multiplying the conditional probability from the c.d.f. times the marginal probability (i.e. $X_{ij} = F(x_i,x_j|x_j)F(x_j)$).

Since criteria interactions (X_{ij}) are, of themselves, events (Ostrofsky, 1977d: 361), they are assigned relative weights (a_{ij}) in the same manner as the marginal criteria (Ostrofsky, 1977d: 374). The requirement still exists that the sum of <u>all</u> relative weights equals 1.0.

Models I-IV assume no interaction among criteria. Therefore, the inclusion of interaction terms results in four additional classes of models (V-VIII) as shown in Figure 21. (Note that our research objective concerns Model VII.) Treating the interaction terms based on probability theory, we have:

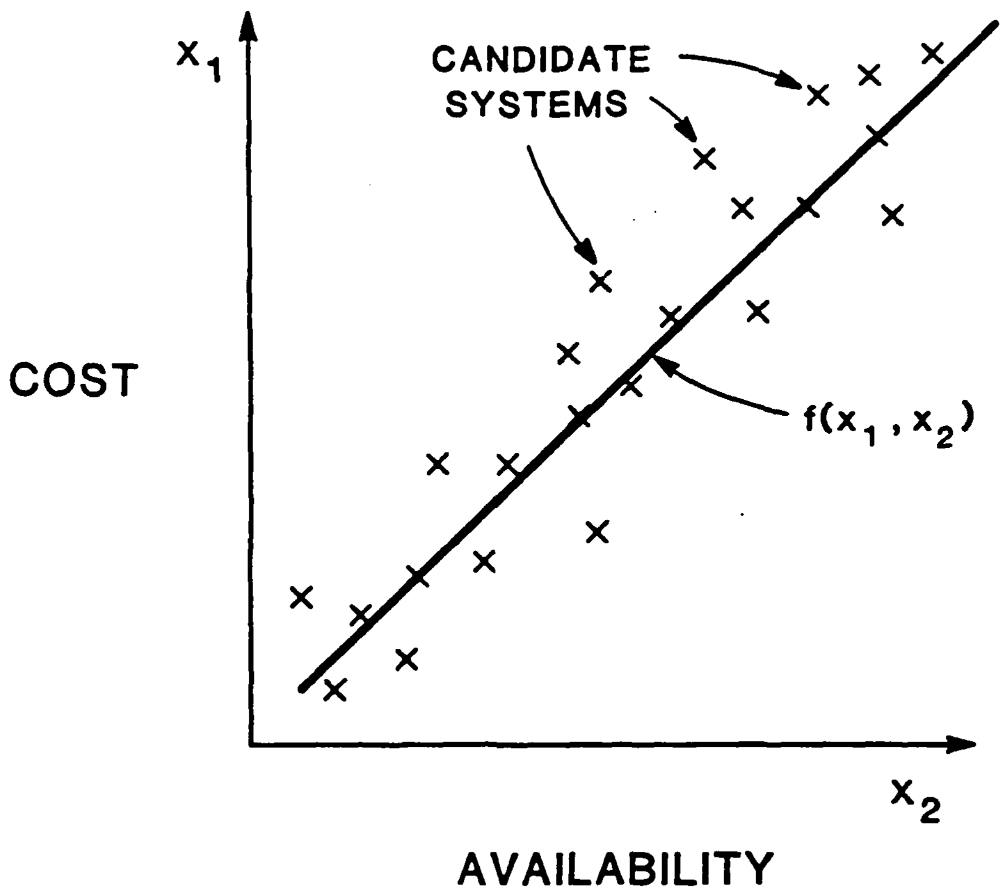


Figure 22. Functional Criteria Interaction

(Simons & Ostrofsky, 1988)

$$
\begin{aligned}
CF = \Sigma a_i X_i &+ \Sigma\delta_{ij}(a_{ij}X_{ij}) \\
&+ \Sigma\delta_{ijk}(a_{ijk}X_{ijk}) \\
&\quad \vdots \\
&+ \Sigma\delta_{ijk\ldots n}(a_{ijk\ldots n}X_{ijk\ldots n}) \qquad \text{Eq. 3.30}
\end{aligned}
$$

where

$$
\delta = \begin{cases} (-1)^s, & \text{when interaction exists and where } s \text{ is the order of interaction when interaction exists (e.g. } X_{ij} \text{ is first order interaction, } X_{ijk} \text{ is second order, etc.)} \\ 0, & \text{when no interaction exists} \end{cases}
$$

Since this approach endeavors to use the laws of probability as a guide, the resulting value of CF should always fall between 0 and 1.0 for each candidate system. Unfortunately, the current criterion function implementation technique may permit CF values less than 0. This indicates that, in some manner, the process used to apply probability theory has been in error.

Summary of Limitations of Existing Decision Approaches

As our description above suggests, perhaps the most difficult step in the design/planning process is the actual selection of a specific candidate system which maximizes the achievement of the specified criteria. Inputs to the decision process are both objective and subjective. Objective inputs include the definition of the candidate systems, their respective subsystems, most parameter values, and quantifiable goals. Subjective inputs include tradeoffs among

conflicting objectives and personal preferences. The primary desired output of the decision process is the identification of the "best" candidate system, where "best" refers to the relative best from among the alternatives considered (although this may or may not be the theoretical "ideal"). In addition, it is frequently desirable to determine a ranking of all candidate systems as well as the relative difference among them.

The literature contains a wealth of approaches (Blin, 1977; Bridgeman, 1922; Cheng and McInnis, 1980; Cohen et al, 1985; Epstein, 1957; Green and Srinivasan, 1978; Harrison, 1975; Holloway, 1979; Ignizio, 1976; Janis and Mann, 1977; Keeney, 1982; Miller, 1970; Roy, 1977; Rubinstein, 1975, 1980, 1986; Sage, 1977; Sakawa and Seo, 1980; Shull, Delbacq, and Cummings, 1970; Simon, 1960, 1978; Tribus, 1969; White and Sage, 1980; Wymore, 1976; Yager, 1981; Zeleney, 1973, 1974, 1976a, 1976b, 1977, 1980, 1981a, 1981b, 1982, 1984) which attempt to model (either prescriptively or descriptively) the human decision making process in such a way that inputs similar to those defined above are converted to some form of useful output. Noteworthy approaches which have been widely cited and/or used include linear programming, goal programming, decision analysis, dimensionless analysis, and design-based techniques. However, each of these methods has demonstrated deficiencies which are summarized in Table 7 and in the paragraphs which follow.

While linear programming and goal programming permit

TABLE 7.

COMPARISON OF DECISION MAKING TECHNIQUES

Approach	Compares many alternatives	Multi objective	Both objective and subjective	Varying relative weights	Optimal vs satisficing	Treats interaction
Linear Programming	P	P	P	N	Y	P
Goal Programming	P	Y	P	N	Y	P
Decision Analysis	Y	Y	P	Y	Y	Y
Dimensionless Analysis	P	Y	P	N	P	N
Tricotyledon Theory	Y	Y	Y	P	P	P
Criterion Function	Y	Y	Y	Y	P	Y

Y - Desired capability is supported

N - Desired capability is not supported

P - Desired capability is partially supported

consideration of multiple objectives, neither method accommodates varying relative weights and both restrict relations among variables to linear combinations and do not explicitly evaluate candidate systems. Even more importantly, both methods produce a theoretically optimal solution resulting from a search of the feasible region of solutions. However, there is no assurance that this solution corresponds to a realizable candidate system.

The perceived potential benefits of decision analysis have not been fully realized in practice (Howard, 1988), perhaps because of a logical contradiction between the theory's intent to descriptively model human preferences and its axiomatic assumption of rationality (Edwards and Luce, 1988). Of more interest in our search for a prescriptive decision making approach is the failure of decision analysis to separate objective considerations (i.e. $\{X_i\}$) from subjective ones (i.e. $\{a_i\}$). Another shortcoming of utility theory techniques is difficulty in translating measures of merit to tangible or measurable characteristics of the candidate systems. This difficulty frequently undermines the confidence of users in the ultimate decision. It is not easy, for example, to convince a company vice president that product design A is preferred to B because it has .037 more utiles than B. Finally, failure to establish the required independence may necessitate a painstaking point-by-point definition of the decision maker's complete preference structure.

While dimensionless analysis distinguishes objective from

subjective considerations and permits direct relation to observable system characteristics, it does not support varying relative weights and its results must be considered heuristic.

Wymore's Tricotyledon approach permits a great deal of flexibility: perhaps too much so. In addition to being potentially cumbersome to apply, there is very little restriction of the manner in which subjective and objective considerations are combined during merit ordering, making the optimality of its solutions arguable.

Finally, most of these techniques fail to account explicitly for interactions among design criteria. Instead, they may require extensive measures to test for independence, require modification of the problem so that no interaction is observed, or simply ignore interactions altogether. (Although linear and goal programming permit specification of interaction as additional constraints, the interaction must be capable of being represented in a linear form. Furthermore, this approach would <u>force</u> the optimal solution to conform to the prevailing observed interaction, even in cases where exceptions might be possible.) The primary exception to this generalization is the criterion function. Unfortunately, some of the past implementations of this approach have obtained results which are not uniformly consistent with the underlying theory.

Specific Research Goal

One could pursue the broad research objective stated in Chapter 1 by attempting to resolve weaknesses in any of the techniques discussed. However, since the criterion function approach contains all the desired characteristics and is directly linked to the design framework, we perceive the most promising alternative to be the identification and resolution of the discrepancies which have produced the observed anomalies in its results. Resolution of these discrepancies would enable the criterion function to achieve the research objective stated in Chapter 1. Therefore, while the overall research objective remains as stated, a key goal to the success of this research will be the clarification of the criterion function implementation technique to produce figures of merit on the range 0 to 1.0, consistent with the underlying theory.

Chapter 4

CRITERION FUNCTION DEVELOPMENT

As a preliminary note, it should be remembered that the criterion function uses probability, not in the context of uncertainty, but rather as a theoretical convention for quantifying performance in such a manner that we are able to: 1) compare candidate systems across criteria, and 2) treat interactions among criteria.

Parzen (1960: 29-30) divides the literature of probability theory into three broad categories: 1) the nature or foundations of probability, 2) mathematical probability theory, and 3) applied probability theory. He describes mathematical probability theory literature as consisting of those writings which are primarily axiomatic. By contrast, the literature of applied probability consists of writings in which probability serves as a tool in scientific or scholarly investigations. Since Ostrofsky's (1968) origination of the use of probability as a basis for the criterion function was a novel application, his work was necessarily concerned with establishing a strong linkage between the axiomatic world of probability mathematics and the criterion function application. By contrast, the current research is able to build on the formal linkage

previously established, focusing more exclusively on its implementation in the criterion function.

Current Method of Criterion Function Synthesis

Prior to recommending changes to the current decision technique, it is necessary to revisit the underlying theory which serves as the basis for the method and the ways in which it is currently applied. This is necessary for two reasons. First, the rationale for the current method must be understood if we are to be able to identify where its methods of implementation have failed. Second, our goal is to more correctly apply the underlying theory of the current approach. Therefore, the theoretical basis for the present approach also serves as the basis for the approach proposed in this research: the difference lies in the clarification of that theory.

Criterion Measurement

The obtainment of raw criterion values for each candidate system through the parameters and submodels was described in Chapters 2 and 3. It was also stated that some method of conversion is necessary to remove the distortions of the raw measurement scales in order to permit comparison across criteria. Folkeson (1982) referred to this as the dimensionality problem.

Two methods of accomplishing this compatibility of scale were

described: linear normalization based on the minimum and maximum values and use of probability theory. Unfortunately, the linear normalization approach suffers from the influence of the original measurement scale in a manner similar to that experienced by the dimensionless analysis approach. Specifically, the results produced are directly influenced by the initial scale of measurement and by the definitions of minimums and maximums. On the other hand, probability theory provides a vehicle for comparison which is strictly relative in nature, i.e. the performance of a candidate system is judged solely in terms of its relation to that of other candidate systems, regardless of the initial scale of measure. In addition, probability theory was selected by Ostrofsky (1968) for use in the criterion function largely because it is the only branch of mathematics which provides an explicit means of treating interactions. The following paragraphs provide a synopsis of the theoretical basis for the use of probability theory in the criterion function. To permit more detailed reference, the most immediately relevant portions of Ostrofsky's work (1977d) have been included as Appendix A.

The Applicability of Probability Theory to Criterion Measurement

Essentially, Ostrofsky (1968, 1977d) defined the existence of a candidate system as an event. The remainder of the universe of events, then, consists of all possible events (candidate systems) within the design space. A function can then be defined which assigns to the occurrence of each event (candidate system) a description

(measure) of that candidate with respect to a specific design criterion. If a set function is then defined over the descriptions in such a way that the axioms of probability theory are satisfied, that set function qualifies as a probability measure.

In other words, Ostrofsky recognized and capitalized on (as has Wymore, 1976) the fact that probability theory is a set of mathematical descriptions and resulting tools which can be applied to virtually any phenomena which meet the required definitions. More explicitly, the use of probability theory is entirely appropriate even in cases where uncertainty, hypothesis testing, or statistical inference are not central issues. The only requirement is that the theory's conventions be adhered to.

In this context, then, the candidate system is considered an event, the performance of the candidate system (for a given criterion) is defined as the relevant description resulting from some function (i.e. criterion modeling), and the descriptions produced can then be acted on by a set function which satisfies probability axioms.

Implementing Criterion Measurement

Based on the rationale just presented, each candidate system's value of the raw criterion measure, x_i, is produced as a function of the observed event within the range $[x_{i\ min}, x_{i\ max}]$. A probability density function (p.d.f.) may then be defined (as the necessary set

function) which acts on the raw criterion scores, x_i, in a manner consistent with probability theory. A cumulative distribution function (c.d.f.) may then be constructed from the p.d.f. However, since our interest lies in the cumulative scores, a c.d.f. is normally constructed directly from the raw criterion scores (Figure 23) without explicit representation of the p.d.f. The c.d.f. then serves as the vehicle for conversion of the raw scores to a unitless value (denoted X_i) on the range [0,1]. Since all criteria have been converted in the same manner, comparability across criteria is now possible. Note that the use of the c.d.f. eliminates all perspective of the absolute quality of a given candidate's performance on a criterion. Instead, all measures become completely relative, serving to identify only how each candidate compares with the remainder of the candidate system set.

Interaction Measurement

The most substantial work concerning interactions within the framework of the criterion function has been that of Ostrofsky (1968, 1977d, 1987) and Folkeson (1982). Ostrofsky's work primarily highlighted the need for treatment of interactions, suggested the use of probability theory as a means for that treatment, and described an approach for measurement of interaction among the criterion (X_i). Folkeson (1982) later termed this type of interaction functional (as opposed to preferential) interaction. In the following paragraphs, we

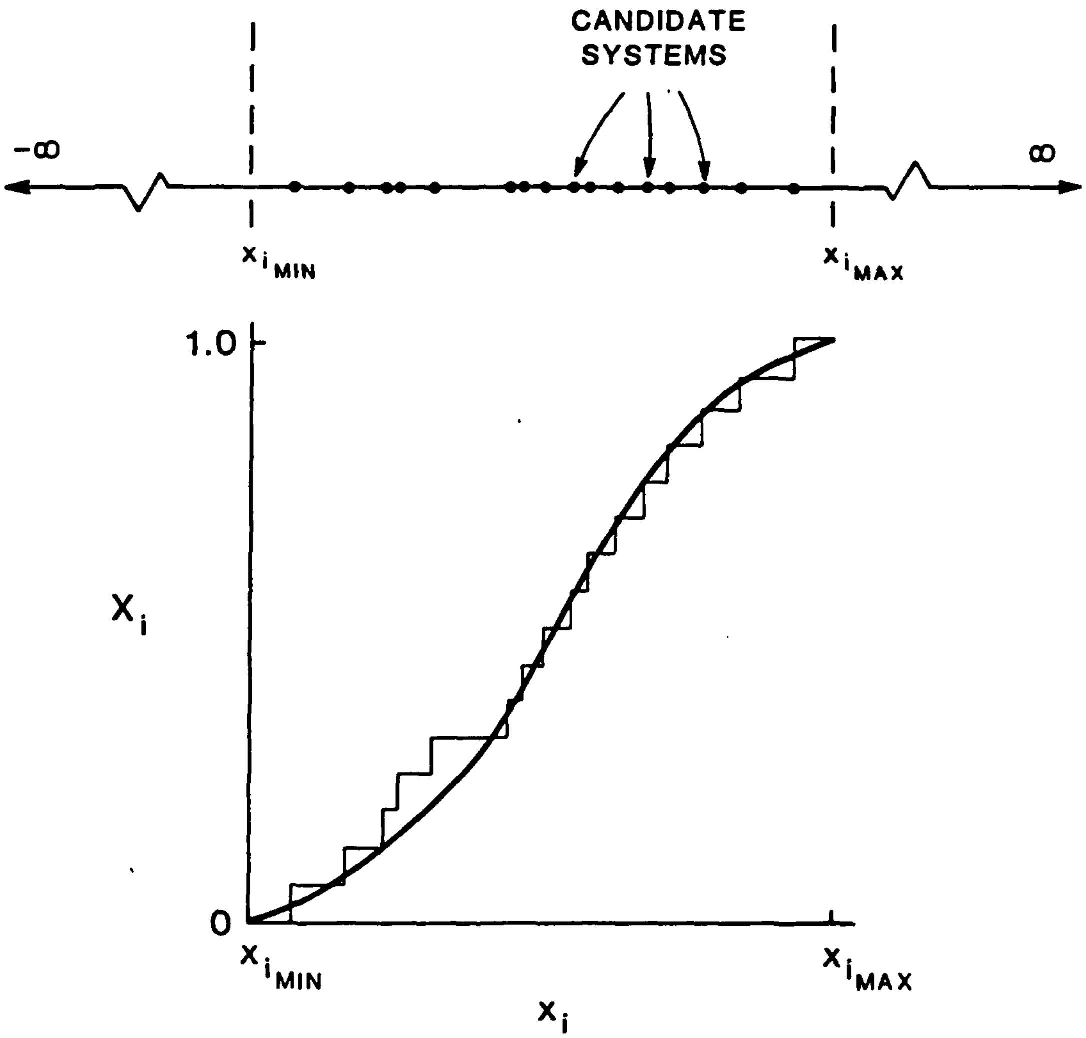


Figure 23. Criterion Cumulative Distribution Function

(Simons & Ostrofsky, 1988)

present Ostrofsky's (1977d, 1987) approach to the use of probability in measuring functional interaction. Since this material is crucially important to our subsequent discussion, we quote directly from Ostrofsky (1977d: 361-362).

(Begin quote)

Functional Interactions as Probabilities

Let

A_i = the event $(X_i < x_i)$
A_{ij} = the event $(X_i < x_i, X_j < x_j)$
.
.
.
$A_{ijk...(J+1)}$ = the event $(X_i < x_i, X_j < x_j, \ldots, X_{J+1} < x_{J+1})$

where

$$J + 1 = n$$

Then

$\delta_i P(A_i)$ = marginal distribution function of the ith criterion:

$$\delta_i = \begin{cases} 1, & \text{when } P(A_i) \text{ exists} \\ 0, & \text{when } P(A_i) \text{ does not exist} \end{cases}$$

$\delta_{ij} P(A_{ij})$ = joint probability or the first-order interaction of x_i and x_j:

$$\delta_{ij} = \begin{cases} 1, & \text{when } P(A_{ij}) \text{ exists} \\ 0, & \text{when } P(A_{ij}) \text{ does not exist} \end{cases}$$

$\delta_{ijk\ldots(J+1)}P[A_{ijk\ldots(J+1)}]$ = joint probability or the jth-order interaction of $x_i, x_j, \ldots, x_{(J+1)}$:

$$\delta_{ijk\ldots(J+1)} = \begin{cases} 1, & \text{when } P[A_{ijk\ldots(J+1)}] \text{ exists} \\ 0, & \text{when } P[A_{ijk\ldots(J+1)}] \text{ does not exist} \end{cases}$$

where the x_is are admissable values of the X_i.

It should be noted that

$$\begin{aligned} P(A_i) &= F(x_i) \\ P(A_{ij}) &= F(x_i, x_j) \\ &\;\;\vdots \\ P[A_{ijk\ldots(J+1)}] &= F[x_i, x_j, \ldots, x_{(J+1)}] \end{aligned} \qquad \text{Eq. 4.1}$$

The concept of the union of events in a probability space has been discussed in Appendix A. When these events are not mutually exclusive they can be shown as (Mood and Graybill, 1963)

$$\begin{aligned} Z &= P\left(\bigcup_{i=1}^{n} A_i\right) \\ &= \sum_{i=1}^{n} \delta_i P(A_i) - \sum_{i}^{n} \sum_{\substack{j \\ i \neq j}}^{n} \delta_{ij} P(A_{ij}) \\ &\quad + \sum_{i}^{n} \sum_{j}^{n} \sum_{\substack{k \\ i \neq j \\ j \neq k \\ i \neq k}}^{n} \delta_{ijk} P(A_{ijk}) \qquad \text{Eq. 4.2} \\ &\quad - \cdots \pm \sum_{i}^{n} \sum_{\substack{j \\ i \neq j \\ \vdots \\ J \neq J+1}}^{n} \cdots \sum_{J+1}^{n} \delta_{ijk\ldots(J+1)} P[A_{ijk\ldots(J+1)}] \end{aligned}$$

Hence the problem of identifying additional candidate systems can be resolved to the problem of identifying a particular value of Z, the union of all events, $A_i | i=1,...,n$. The degenerate case exists where the x_i are mutually exclusive and

$$Z = P(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i) \qquad \text{Eq. 4.3}$$

(End quote)

Measuring (Computing) Functional Interactions

Let x_1 and x_2 be two design criteria with a density function $f(x_1,x_2)$. From laboratory testing or other means the points along $g(x_1,x_2)$ can be obtained (Figure 24a). (Note that $g(x_1,x_2)$ is a two-dimensional projection of the density function, $f(x_1,x_2)$.) These points are then plotted and the Kolmogorov-Smirnov test used to fit the theoretic distribution $F(x_1,x_2|x_1)$ (Figure 24b). Based on Bayes's Theorem, the interaction term is then computed as

$$X_{12} = F(x_1,x_2) = F(x_1,x_2|x_1)*F(x_1) \qquad \text{Eq. 4.4}$$

For the second-order interaction, the value of the third criterion resulting from the combination of the first two is plotted as shown in Figure 25a. In this manner, $h(x_1,x_2,x_3)$ is transformed

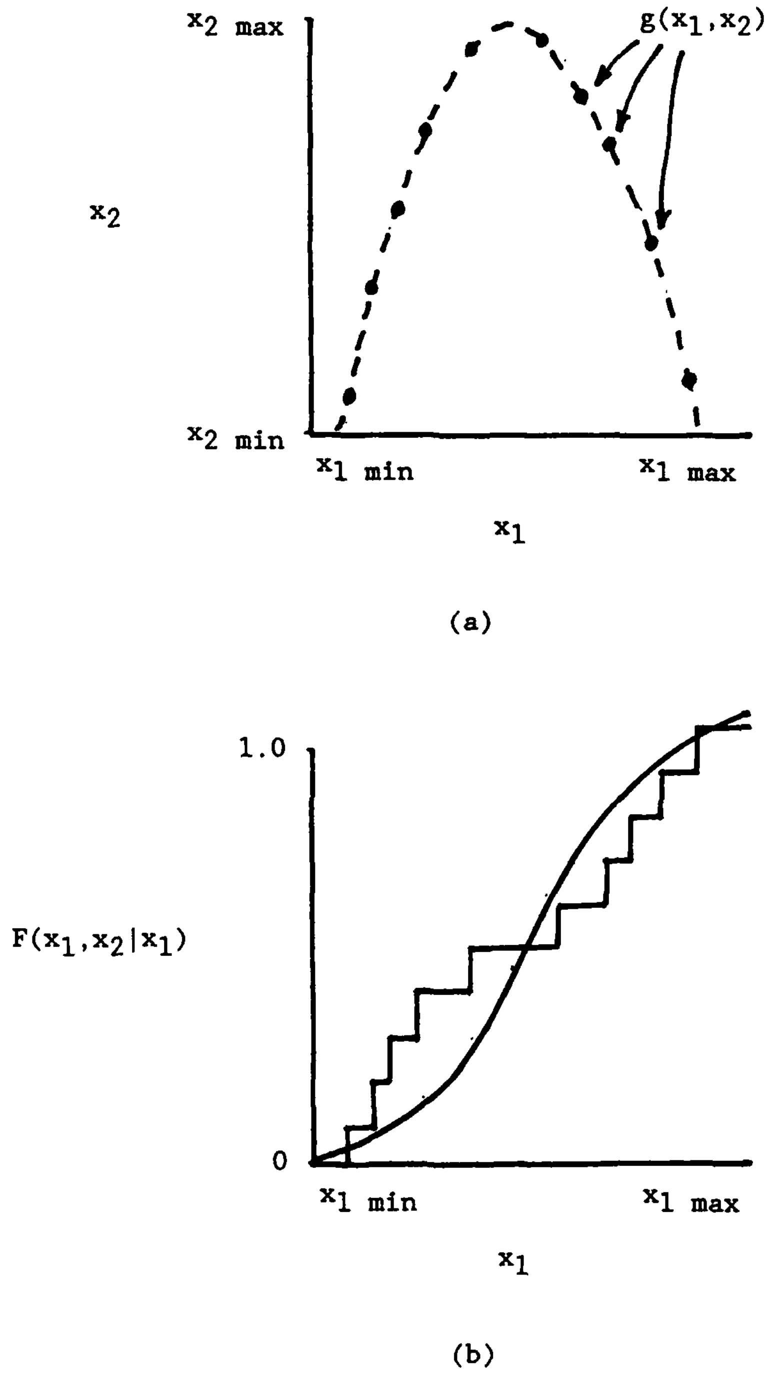
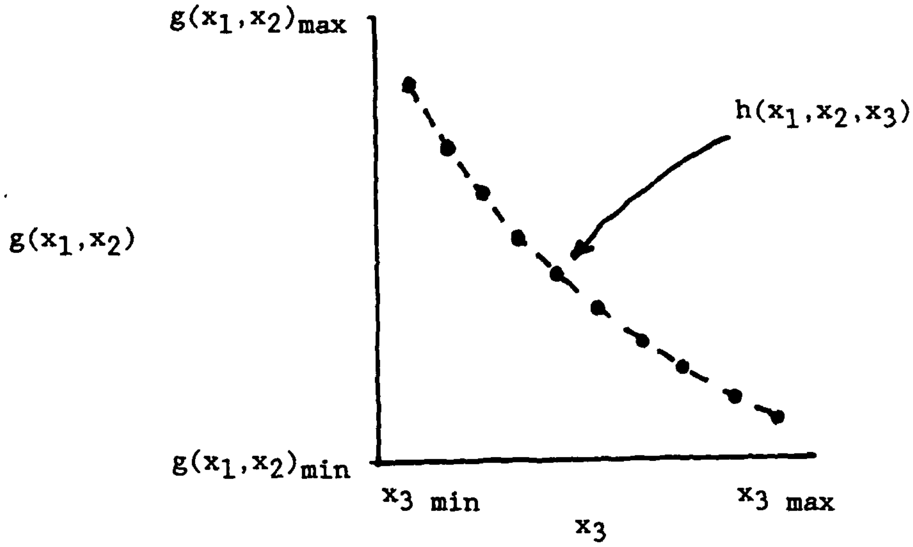


Figure 24. Quantitative Description of Functional Interaction

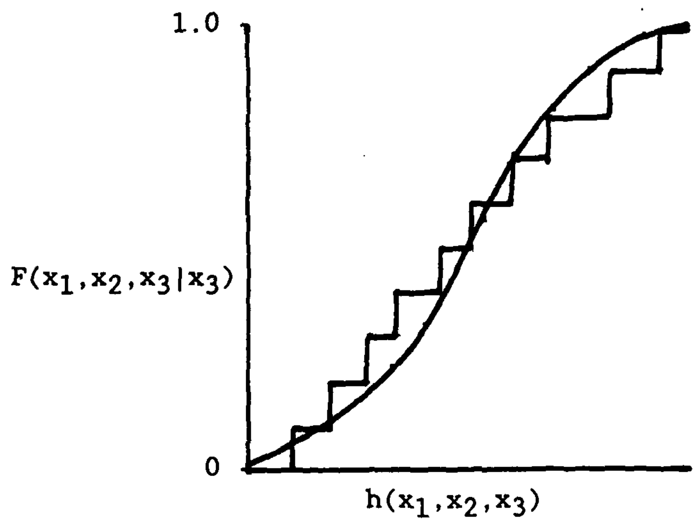
a. Determination of $g(x_1,x_2)$

b. Conversion to Cumulative Distribution Function

(Ostrofsky, 1977d)



(a)



(b)

Figure 25. Second Order Interaction

a. Determination of $h(x_1,x_2,x_3)$

b. Conversion to Cumulative Distribution Function

(Ostrofsky, 1977d)

into the probability space as shown in Figure 25b. This procedure is continued until all interactions have been transformed into probabilities.

Inclusion of Relative Weights

In the following paragraphs, we review the three major developments in the inclusion of relative weights: the basic role of relative weights within the context of probability theory (Ostrofsky, 1968, 1977d), the possibility and treatment of varying relative weights (Ostrofsky, 1977d, 1987), and the concept of preferential independence (Folkeson, 1982).

Relative Weights as Probabilities

In the previous section it was shown that the union of events which are not mutually exclusive results in Equation 4.2, where the probabilities are a function of the criteria values resulting from a given candidate system. When the relative weight, a_i, of each criterion is included, the result will be a real number which, when obtained for each candidate in the set of candidate systems for the design, defines the hierarchy of the desirability of the candidates by establishing the list of candidates with the highest values at the top of the hierarchy.

A raw measure of relative value, a'_i, can be transformed to a

subjective probability, a_i (Fishburn, 1970). Ostrofsky observes that when $A_i = P[X_i < x_i]$, an examination of the a_i and $P(A_i)$ indicates that, in the context of design, the a_i are statistically independent of $P(A_i)$ since a_i will remain constant for any value of $P(A_i)$ within the given interval of x_i; hence

$$a_i \cap P(A_i) = a_i P(A_i) = \theta_i \qquad \text{Eq. 4.5}$$

Recall that, as we have defined it, $P(A_i)=F(x_i)$ so that $\theta_i = a_i F(x_i)$. Then, when the x_i are not mutually exclusive, Equation 4.2 may be revised as follows,

$$\begin{aligned}
CF &= P\left(\bigcup_{i=1}^{n} a_i A_i\right) \\
&= \sum_{i=1}^{n} \delta_i \theta_i - \sum_{\substack{i \\ i \neq j}}^{n} \sum_{j}^{n} \delta_{ij} \theta_{ij} \\
&\quad + \sum_{\substack{i \\ i \neq j \\ j \neq k \\ i \neq k}}^{n} \sum_{j}^{n} \sum_{k}^{n} \delta_{ijk} \theta_{ijk} \qquad \text{Eq. 4.6} \\
&\quad - \ldots \pm \sum_{\substack{i \\ i \neq j \\ \vdots \\ J \neq J+1}}^{n} \sum_{j}^{n} \ldots \sum_{J+1}^{n} \delta_{ijk\ldots(J+1)} \theta_{ijk\ldots(J+1)}
\end{aligned}$$

where

$$\delta_i \theta_i = \delta_i a_i P(A_i) = \delta_i a_i F(x_i) \qquad \text{Eq. 4.7}$$

Equation 4.7 may be described as the weighted marginal probability of the ith criterion, where

$$\delta_i = \begin{cases} 1, & \text{when } \theta_i \text{ exists} \\ 0, & \text{when } \theta_i \text{ does not exist} \end{cases}$$

Similarly,

$$\delta_{ij}\theta_{ij} = \delta_{ij}a_{ij}P(A_{ij}) = \delta_{ij}a_{ij}F(x_i,x_j) \quad \text{Eq. 4.8}$$

This is the weighted value of the first-order interaction of the x_i and x_j as measured by their joint marginal distribution where

$$\delta_{ij} = \begin{cases} 1, & \text{when } \theta_{ij} \text{ exists} \\ 0, & \text{when } \theta_{ij} \text{ does not exist} \end{cases}$$

Likewise,

$$\delta_{ijk}\theta_{ijk} = \delta_{ijk}a_{ijk}P(A_{ijk}) = \delta_{ijk}a_{ijk}F(x_i,x_j,x_k) \quad \text{Eq. 4.9}$$

This term is the weighted value of the second-order interaction of x_i, x_j, and x_k as measured by their joint marginal distribution where

$$\delta_{ijk} = \begin{cases} 1, & \text{when } \theta_{ijk} \text{ exists} \\ 0, & \text{when } \theta_{ijk} \text{ does not exist} \end{cases}$$

Therefore, the general form for any term in Equation 4.6 becomes

$$
\begin{aligned}
\delta_{ijk\ldots(J+1)}\theta_{ijk\ldots(J+1)} \\
&= \delta_{ijk\ldots(J+1)} a_{ijk\ldots(J+1)} P[A_{ijk\ldots(J+1)}] \\
&= \delta_{ijk\ldots(J+1)} a_{ijk\ldots(J+1)} F[x_i, x_j, x_k, \ldots, x_{(J+1)}]
\end{aligned}
\qquad \text{Eq. 4.10}
$$

Equation 4.6 then defines the relative value of a candidate system to include both the relative importance of the respective criterion, x_i, and the value of the probability density of the x_i.

Varying Relative Weights

It is entirely reasonable (Ostrofsky, 1977d: 372) for relative weights to assume different values at different points on the range $(x_{i\,min}, x_{i\,max})$. In other words, a situation may exist where one set of $\{a_i\}$ exists for $\{x_i\}$ when

$$x_{i\,min} \le x_i \le x'_i, \quad i=1,\ldots,n \qquad \text{Eq. 4.11}$$

and another set of $\{a'_i\}$ exists when

$$x'_i \le x_i \le x''_i, \quad i=1,\ldots,n \qquad \text{Eq. 4.12}$$

and a third set of $\{a''_i\}$ when

$$x''_i \le x_i \le x'''_i, \quad i=1,\ldots,n \qquad \text{Eq. 4.13}$$

and so on until there is a value of a_i for all intervals in the complete range of each x_i (see Table 8). It was this possibility

TABLE 8.

RELATIVE WEIGHTS VARYING BY INTERVAL

(Ostrofsky, 1977d)

	Interval				
	I	II	III	...	$\epsilon+1$
	$x_{i\,min} \le x_i \le x_i'$	$x_i' < x_i \le x_i''$	$x_i'' < x_i \le x_i'''$	...	$x_i^{\epsilon} < x_i \le x_{i\,max}$
a_1	a_1'	a_1''	a_1'''	...	a_1^{ϵ}
a_2	a_2'	a_2''	a_2'''	...	a_2^{ϵ}
a_3	a_3'	a_3''	a_3'''	...	a_3^{ϵ}
.	.	.	.		.
.	.	.	.		.
.	.	.	.		.
a_n	a_n'	a_n''	a_n'''	...	a_n^{ϵ}
	$\sum_{i}^{n} a_i' = 1$	$\sum_{i}^{n} a_i'' = 1$	$\sum_{i}^{n} a_i''' = 1$	...	$\sum_{i}^{n} a_i^{\epsilon} = 1$

which led to the proposed existence of the eight decision models shown in Chapter 3 of this dissertation. However, Ostrofsky observes (1977d: 372) that, "the properties of a probability density apply to the $\{a_i\}$ for each predefined interval in the range of $\{x_i\}$." Therefore,

$$\sum_{i}^{n} a_i = 1 \qquad \text{Eq. 4.14}$$

for each interval although the a_i do not necessarily sum to 1 over the total range of a given x_i.

When more than one interval exists, Ostrofsky states that the relative weights must be compared within the same interval, where transitivity holds across criteria (as shown in Table 8). This is intended to support a direct comparison of the desirability of candidate systems by comparing their resulting real numbers in [0,1].

Given any set of relative weights which vary across intervals, Ostrofsky (1987) has suggested a two-step normalization scheme for computation of the relative weights which insures Equation 4.14 remains satisfied. This scheme consists of vertical and horizontal normalization of the a_i, where the terms vertical and horizontal refer to the orientation of Table 8.

Vertical normalization means that the relative weight for the kth

criterion in the β interval ($a_k{}^{\beta}$) is first divided by the sum of all n $a_i{}^{\beta}$ in the β interval. When a finite number of intervals have been defined, this normalization can be accomplished once prior to evaluating candidate systems so that all relative weights in the same interval already satisfy Equation 4.14 as in Table 8. Horizontal normalization is then accomplished by insuring that Equation 4.14 is satisfied for each candidate system. This is achieved by dividing each a_i for each candidate system by the sum of the a_i for that candidate system.

Model V (constant relative weights with interactions) was implemented by Peschke (1986) in optimizing an information resource management system. The case (just described) of relative weights varying throughout the [0,1] range but remaining constant within each defined interval is accommodated by Models II and VI, depending on the presence of interactions. Model VI was demonstrated by Folkeson for the design of a piece of aircraft support equipment. A logical extension of this case is the scenario in which relative weights vary continuously throughout the X_i range (Model VII). This may be thought of as an extension of Models II and VI in which there are an infinite number of intervals. This model has been suggested (Folkeson, 1982; Peschke, 1986; Ostrofsky, 1987) but not previously demonstrated.

Preferential Independence

As mentioned earlier, Folkeson (1982) addressed two forms of

interaction: functional (interaction among the X_i) and preferential (interaction among the a_i). Although he suggested a variety of methods for identifying potential functional interaction (e.g. designer suspicions, correlation coefficients, regression, etc.), he provided no specific suggestions for treating those interactions and the example he described had no functional interactions which he was able to quantify. However, by drawing on the field of decision analysis, Folkeson directly addressed the treatment of the lack of preferential independence. In the following paragraphs, we paraphrase the most significant portion of this contribution.

As stated during the discussion of decision analysis in Chapter 3, the presence of interaction among criteria may lead to the need for complete multi-dimensional specification of the decision maker's preference structure. According to Keeney and Raiffa (1976), the pair of attributes X and Y is <u>preferentially independent</u> of Z if the conditional preferences in (X,Y) space given Z' do not depend on Z'.

For example,

$$(X_1, Y_1, Z) > (X_2, Y_2, Z), \text{ for all } Z$$

Keeney and Raiffa (1976:105) state that the additive form is appropriate only when all attributes are mutually preferentially independent:

A value function v may be expressed in an additive form

$$v(X, Y, Z) = v_x(X) + v_y(Y) + v_z(Z)$$

where v_x, v_y, and v_z are single attribute value functions, if and only if {X, Y} is preferentially independent of Z, {X, Z} is preferentially independent of Y, and {Y, Z} is preferentially independent of X.

Subsequent work by Dyer and Sarin (1979) and by Dewispelare and Sage (1979) showed the multiplicative form was appropriate, even in the presence of interaction, as long as the attributes are mutually weak difference independent (WDI). Folkeson (1982) describes WDI as follows:

Define the symbol $X_{\bar{i}}$ to indicate all components of X not contained in X_i. Then, criterion X_i is WDI of $X_{\bar{i}}$ if given any b_i, c_i, d_i, $e_i \in X_i$, such that

$$v(b_i, b_{\bar{i}}) - v(c_i, b_{\bar{i}}) \geq v(d_i, b_{\bar{i}}) - v(e_i, b_{\bar{i}}) \qquad \text{Eq. 4.15}$$

for some $b_{\bar{i}} \in X_{\bar{i}}$, then it is required that

$$v(b_i c_{\bar{i}}) - v(c_i c_{\bar{i}}) \geq v(d_i c_{\bar{i}}) - v(e_i c_{\bar{i}}) \qquad \text{Eq. 4.16}$$

for any $c_{\bar{i}} \in X_{\bar{i}}$.

As the number of criteria present in a given decision problem increase, the proof of WDI can become very tedious. The verification of the appropriateness of the multiplicative form of the criterion function requires checking all subsets of criteria for WDI. Dewispelare and Sage (1979) developed a theorem to make this task simpler and less time-consuming.

Theorem: Given criteria $X_1, X_2, \ldots, X_n$, the following are equivalent.

a) Criteria $X_1, X_2, \ldots, X_n$ are mutually weak difference independent (MWDI).

b) X_i is weak difference independent of $X_{\bar{i}}$, and (X_i,X_j), $j \neq i$, is preferentially independent (PI); $j=1,2,3,\ldots,n$, $n \geq 3$.

Folkeson indicates that preferential independence should have been checked before considering the additive model in the first place. He therefore concludes that checking for WDI in X_i and $X_{\bar{i}}$ is all that is required to proceed in using the following multiplicative form of the criterion function:

$$CF_x = \sum_{i=1}^{n} a_i X_i + K \sum_{\substack{i=1 \\ j>i}}^{n} a_i a_j X_i X_j + K^2 \sum_{\substack{i=1 \\ j>i \\ k>j}}^{n} a_i a_j a_k X_i X_j X_k + \ldots + K^{n-1} \prod_{i=1}^{n} a_i X_i \qquad \text{Eq. 4.17}$$

or

$$1 + K(CF_x) = \prod_{i=1}^{n} [1 + K a_i X_i] \qquad \text{Eq. 4.18}$$

He further observes that if the X_i are preferentially independent and the a_i sum to 1.0, the additive criterion function is appropriate (i.e. $K = 0.0$). However, if the a_i do <u>not</u> sum to 1.0, the multiplicative form of the criterion function should be used with the scaling constant K derived from the a_i values using the following relation

$$1 + K = \prod_{i=1}^{n} (1 + Ka_i) \qquad \text{Eq. 4.19}$$

The correct value of K is obtained through convergence by repeatedly solving Equation 4.19. When the sum of the a_i is greater than 1.0, K will fall between -1.0 and 0.0. When the sum of the a_i is less than 1.0, K will be greater than 0. (See Keeney and Raiffa, 1976:307,324-325,347-348; and Sage, 1977:246-250.)

Folkeson suggests that sensitivity analysis should again be performed at this stage to provide the decision maker with not only discrete value measures but also a feel for the marginal effects in the area of the optimal candidate or around other points of interest.

Note that Folkeson identified the lack of preferential independence by the failure to satisfy Equation 4.14. Recall, however, that the vertical and horizontal normalization scheme of Ostrofsky (1986) will insure that this relationship is always satisfied. Clearly, then, two different schools of thought exist in this regard. The distinction between these approaches probably lies in their fundamental perception of the significance of the a_i and in particular, the interaction relative weights (e.g. a_{ij}). We perceive this to be an important issue which should be pursued in future research. However, since pursuit of this distinction is beyond the scope of the present research, we will henceforth employ the double

normalization scheme of Ostrofsky, which has been applied in practice to date.

Revised Approach to Criterion Function Synthesis

Before proceeding with a detailed re-evaluation of the methods just reviewed, it is important to reexamine the intuitive or logical basis for the form of the criterion function itself. In so doing, we seek to avoid confusion concerning the sometimes subtle meaning of apparently obvious concepts.

By way of justifying this discussion, consider Bertrand's paradox (Parzen, 1960: 302). The paradox begins with the following problem:

> Let a chord be chosen randomly in a circle of radius r. What is the probability that the length X of the chord will be less than the radius r?

This problem is capable of producing a variety of different answers, each seemingly quite logical, yet varying in value from 0.134 to 0.333. The paradox is caused by a lack of clarity concerning what is meant by a randomly chosen chord. Each of the different answers is either correct or incorrect depending on which interpretation of the problem is accepted at the outset. From this paradox we conclude that, even for seemingly well-defined events, a thorough reexamination of the meaning of that event may shed significant light on how we compute the solution. Therefore, we proceed by focusing on the intuitive meaning of the event(s) of interest.

Logical Basis for the Form of the Criterion Function

Parzen (1960: 12) defines an event as a set of descriptions. "To say that an event E has occurred is to say that the outcome of the random situation under consideration has a description that is a member of E." In the context of design decision making, each candidate system may be seen as an event whose descriptions are stated in terms of parameter values and the resulting criteria values.

Define ϵ_i as the event that the candidate system being evaluated is preferred to another candidate system with respect to the ith criterion. Define θ_i as the probability associated with event ϵ_i, i.e. $\theta_i = P[\epsilon_i]$. (Note: For the moment, we will not attempt to separate the constituent elements of performance, X_i, and relative importance, A_i. By stating that a candidate system is preferred, we temporarily accept that this description includes consideration of both performance and relative importance. However, we will make a distinction between the two in the sections following this more general discussion.)

In simple terms, what we ideally want is the candidate system which has the greatest value of θ_i for all criteria simultaneously. In other words, we want the candidate system which outperforms all other candidate systems on each and every criterion. Such a candidate system, if it exists, would be said to dominate the other candidate systems (Zeleney, 1977). In the case where such a candidate system

exists, the decision is trivial since the dominating candidate system is clearly superior in all respects.

The challenge, then, is how to decide when no candidate system exists which is dominant with respect to all criteria simultaneously. One way to make such a decision is to try to minimize the probability of making a bad choice. In our context, the worst possible choice would be the selection of a candidate system which is inferior on all criteria simultaneously. In terms of ϵ_i and θ_i, such a choice would have the lowest values of θ_i relative to the other candidate systems. Alternatively, we may define ϵ_i^c as the complement of ϵ_i: the event that the candidate system being evaluated is not preferred to another candidate system on the ith criterion. Similarly, define θ_i^c as the probability associated with ϵ_i^c. Then, for a bad choice, values of θ_i^c would be relatively high since $P[\epsilon_i^c]=1-P[\epsilon_i]$ and, therefore, $\theta_i^c=1-\theta_i^c$ (Parzen, 1960: 19).

Stated in this manner, our goal would be to find that candidate system which has the smallest values of θ_i^c for all i simultaneously. However, due to the reciprocal nature of complementary events, no such candidate system exists or we would have identified it initially as dominating all other candidate systems. Therefore, a reasonable alternative goal is to come as close to this ideal as possible: to seek that candidate system which is least likely to be a bad choice, i.e. that candidate system which has the smallest simultaneous (across all i) combination of θ_i^c.

According to Parzen (1960: 13), the intersection (EF) of two events "is defined as consisting of the descriptions that belong to both E and F; consequently, the event EF is said to occur if and only if both E and F occur, which is to say that the observed outcome has a description that is a member of both E and F." The simultaneous occurrence of the $\epsilon_i{}^c$ is, therefore, defined as their intersection since the collection of $\epsilon_i{}^c$ for a given candidate system constitutes the description of that candidate system in terms of each i. A candidate system is characterized by simultaneous descriptions with respect to all criteria. Therefore, we are interested in finding that candidate system which has the minimum value of the probability of the intersection of its $\epsilon_i{}^c$.

Parzen (1960: 12) would denote the intersection of the $\epsilon_i{}^c$ as $\epsilon_1{}^c\epsilon_2{}^c\ldots\epsilon_n{}^c$ where n is the number of criteria. It is from this representation that the intuitive basis for the form of the criterion function emerges.

According to de Morgan's laws (Parzen, 1960: 16), for n events, $E_1, E_2, \ldots, E_n$,

$$(E_1 \cup E_2 \cup \ldots \cup E_n)^c = E_1{}^cE_2{}^c\ldots E_n{}^c \qquad \text{Eq. 4.20}$$

Therefore, we have that

$$\epsilon_1{}^c\epsilon_2{}^c\ldots\epsilon_n{}^c = (\epsilon_1 \cup \epsilon_2 \cup \ldots \cup \epsilon_n)^c \qquad \text{Eq. 4.21}$$

In other words, in order to avoid a bad choice (when no candidate

system dominates), we wish to minimize the probability of the event $(\epsilon_1 \cup \epsilon_2 \cup \ldots \cup \epsilon_n)^c$, i.e.

$$\mathrm{Min}\ (\theta_1 \cup \theta_2 \cup \ldots \cup \theta_n)^c \qquad \text{Eq. 4.22}$$

Due to the nature of complementary events, we have (Parzen, 1960: 19)

$$\mathrm{Min}\ (\theta_1 \cup \theta_2 \cup \ldots \cup \theta_n)^c \dashrightarrow \mathrm{Min}\ 1-(\theta_1 \cup \theta_2 \cup \ldots \cup \theta_n) \qquad \text{Eq. 4.23}$$

$$\dashrightarrow \mathrm{Max}\ (\theta_1 \cup \theta_2 \cup \ldots \cup \theta_n) \qquad \text{Eq. 4.24}$$

If the ϵ_i are mutually exclusive, then

$$\mathrm{Max}\ (\theta_1 \cup \theta_2 \cup \ldots \cup \theta_n) = \mathrm{Max}\ (\theta_1 + \theta_2 + \ldots + \theta_n) \qquad \text{Eq. 4.25}$$

However, for two events to be mutually exclusive, one event must be able to occur without the occurrence of the other. Therefore, in the context of design decision making, to permit mutually exclusive θ_i is to permit the evaluation of candidate systems which do not have descriptions (or observed behavior) on all criteria. While this may be theoretically possible, historical application of the criterion function has been restricted to candidate systems which have descriptions (measurements) for <u>all</u> criteria. In other words, the ϵ_i are <u>not</u> mutually exclusive. On the contrary, the existence of a candidate system infers the presence of ϵ_i for all i. (This assumption is clearly embedded in Ostrofsky's original work (1968, 1977d) as well.) We consider this a reasonable assumption and accept it for the balance of this research. In the worst case, the assumption serves only to limit, but not invalidate, our conclusions.

When two events are <u>not</u> mutually exclusive, their union is computed (Parzen, 1960: 19) as

$$P[E \cup F] = P[E] + P[F] - P[EF] \qquad \text{Eq. 4.26}$$

where P[EF] is the probability associated with the intersection EF, as defined previously (Parzen, 1960: 13). For the case of two criteria, then, we have that

$$P[\epsilon_1 \cup \epsilon_2] = P[\epsilon_1] + P[\epsilon_2] - P[\epsilon_{12}] \qquad \text{Eq. 4.27}$$

where ϵ_{12} is the intersection of events ϵ_1 and ϵ_2. Equivalently,

$$(\theta_1 \cup \theta_2) = \theta_1 + \theta_2 - \theta_{12} \qquad \text{Eq. 4.28}$$

For the more general case of n criteria, Equation 4.28 is extended as

$$(\theta_1 \cup \theta_2 \cup \ldots \cup \theta_n) = \sum_{i=1}^{n} \theta_i - \sum_{i=1}^{n} \sum_{j=i+1}^{n} \theta_{ij} \pm \cdots \pm \sum_{i=1}^{n} \cdots \sum_{m=l+1}^{n} \theta_{i \ldots m} \qquad \text{Eq. 4.29}$$

where all θ with more than one subscript represent the probabilities of the intersections of the associated events.

Note that the form of the relationship just derived is precisely that of the criterion function (Eq. 4.6). Our goal is to find the candidate system which maximizes that expression. In so doing, the preceding discussion reveals that we are using the relationships of probability theory to make the best possible decision: that which minimizes the probability of being inferior.

The sections which follow extend the reexamination begun here by analyzing the effects of this discussion on the constituent elements of the θ: the X_i, the A_i, and the intersection terms. These sections will identify some inconsistencies between the current method of

criterion function implementation and the underlying theory. The inconsistencies are a reflection of the limitations of the current methods of implementation and are not a condemnation of the criterion function approach in general or the work of previous researchers.

The intent of the proposed research is to suggest and demonstrate revisions to the current methods of implementation which will resolve the inconsistencies. These revisions will be based on the rediscovery of simple truths. The rediscovery process will require that we back up and correctly apply what had previously been discovered to be true but has somehow been overlooked in the implementation process over time, and will in itself demonstrate, not only increased knowledge and awareness of the process, but also effective research.

Reexamination of Criterion Measurements

In the following sections we focus our general discussion of the θ_i to the specific subcategory of the X_i.

Linear Normalization vs. Probability

The first elementary issue in reexamining the synthesis of the criterion function is a reexamination of the basic approach to characterizing candidate system performance with respect to a given design criterion. Given a set of raw (observed) criterion performance measurements, two methods of conversion to permit unitless comparison

have been suggested thus far: linear normalization using $x_{i\ min}$ and $x_{i\ max}$ and conversion to probabilities via cumulative distribution frequencies (c.d.f.). Let us briefly reconsider the strengths and weaknesses of the two approaches.

The primary distinguishing feature of linear normalization is that the resulting measures attempt to reflect an absolute value of performance. By this we mean that the normalized value, X_i, reflects the performance of a candidate system solely with respect to the constant range $[x_{i\ min}, x_{i\ max}]$. If a candidate system's raw performance exceeds 50% of the defined criterion range, its score will be .5, <u>regardless</u> of how well the other candidate systems perform. This has the advantage of reflecting dramatic differences in the performance of candidate systems in terms of an absolute benchmark scale.

By contrast, the c.d.f. approach to criterion scoring is completely relative. Regardless of where a candidate's raw score falls in the $[x_{i\ min}, x_{i\ max}]$ range, its criterion performance will be computed strictly as a function of how that candidate compared to all others observed. Therefore, even minute changes in the absolute performance of a candidate may cause a dramatic change in X_i if the candidates are closely grouped.

However, a significant drawback of the normalization approach is its strict dependence on the definitions of $x_{i\ min}$ and $x_{i\ max}$. Given

that comparison across criteria will ultimately serve as the basis for selection, the determination of the possible ranges of the criteria may clearly influence the decision. Take, for example, the trivial case of two candidate systems and two equally important criteria as shown in Table 9.

When $x_{2\ max}$ changes from 10 to 100, the overall decision changes from favoring candidate A to candidate B. While this characteristic may be desirable where clearly defined and meaningful minimums and maximums exist, it is an undesirable trait in the more common case where the ranges are speculative or arbitrary. By contrast, the probability based approach is completely insensitive to such range changes.

Clearly then, the use of probability provides a better means for assessing <u>relative</u> performance, which is, in fact, the crux of the design decision. Recall that the design decision is concerned with the identification of the candidate system which is relatively best from among the set considered. Note that the determination of how much better one candidate is than another will be determined by the shape of the c.d.f. curve. As described so far, the shape of the c.d.f. will be determined by the performance of the defined set of candidate systems. However, if one is inclined to assume that the performance distribution of the defined set is representative of the distribution of the theoretic exhaustive set, then the relative performance of a candidate system is effectively assessed relative to

TABLE 9.

EXAMPLE CANDIDATE SYSTEM CF SCORES

Candidate	x_1	x_2	CF based on x_1, x_2 ranges: x_1=[0,10] x_2=[0,10]	x_1=[0,10] x_2=[0,100]
A	4	9	.65*	.245
B	5	1	.30	.255*

where $CF = .5X_1 + .5X_2$

* Preferred candidate

all possible candidate systems including those which are, as yet, undefined.

The implications of assumptions concerning the shape of the c.d.f. can be carried a step further. Assume for a moment that we do not consider the performance distribution of the defined set of candidates to be representative of the theoretic exhaustive set but that we have some reason to suspect some other, specific distribution. In this case, it seems entirely appropriate to utilize the suspected theoretical distribution to construct the c.d.f. and produce the X_i.

In summary then, we have observed that unlike the linear normalization approach, the use of probability provides a strictly relative measure which is insensitive to arbitrary definitions of the raw criteria ranges. In addition, we have said that the relationships observed among candidates using the probability based approach may be generalized to unspecified candidates if we consider the observed distribution representative of a broader case. On the other hand, we suggest that known or suspected distribution characteristics of the complete (theoretical) set of alternatives may shed new light on those candidates we have defined. Finally, we recall that probability provides a theoretic basis for treating interactions.

Probability as a Measure of Dominance

For the reasons described above e accept the use of probability

theory as the preferred basis for comparison across criteria. As shown earlier in this chapter, Ostrofsky (1968, 1977d) has rigorously established the application of that theory in the context of the design decision criteria. With respect to measurement of marginal criteria values, we indorse that contribution and provide the following amplifying comments.

Recall that the probability based measure of the performance of a candidate system with respect to a given criterion is defined as $F(x) = P\{\gamma : X(\gamma) < x\}$. Verbally, this may be stated as follows: <u>the performance of a candidate system with respect to a given criterion is the probability that the raw performance of the candidate being evaluated exceeds that of another candidate system on the criterion of interest</u>. Those candidates whose performance is exceeded may be described as being dominated with respect to the ith criterion by the system being evaluated. Therefore, X_i effectively measures <u>the portion of the set of candidate systems which is dominated on the ith criterion by the candidate system being evaluated</u> (Keeney and Raiffa, 1976; Zeleney, 1977). Graphically, this is shown in Figure 26.

Measuring Intersections and Interaction

<u>Interaction vs. Intersection</u>

If, as we have asserted in the previous section, the significance and computation of the X_i are appropriate and correct, then the

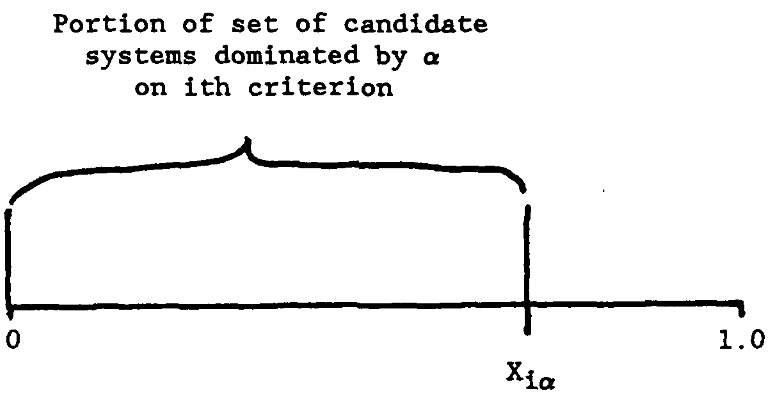
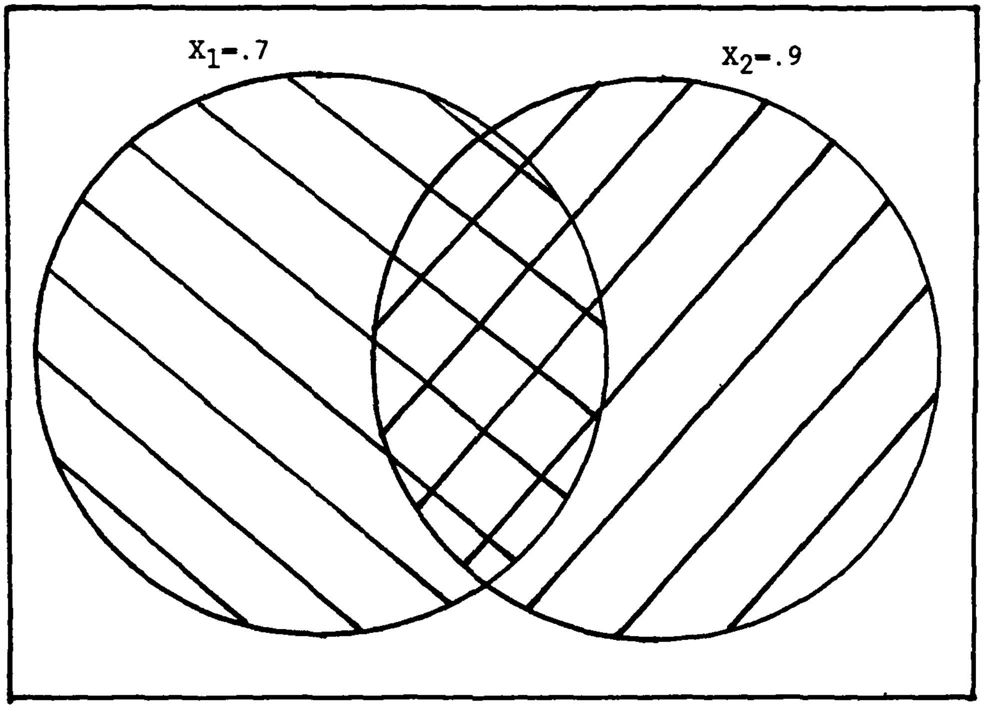


Figure 26. Single Criterion Dominance

source of the unexplained results (i.e. CF<0) must lie in the computation of the interaction terms, the relative weights, or both. Indeed, we will suggest that this is precisely the case. In this section, we consider what Folkeson (1982) termed functional interactions, namely the X_{ij}. That aspect of interaction characterized by Folkeson as preferential interaction will be discussed in the subsequent section on the inclusion of relative weights.

To begin our discussion, we recall our earlier examination of the form of the criterion function as expressed in Equation 4.6. Clearly, this equation is the generic representation of the computation of the probability of the <u>union</u> of the n marginal events (Ostrofsky, 1977: 362). Consequently, we see that <u>the terms heretofore referred to as interaction terms are more accurately the probabilities of the intersections of the various marginal events</u>. Visually, the Venn diagram (e.g. Figure 27) provides a useful means for recognizing the unions and intersections of events. The intersection of events A and B is the event that <u>both</u> A <u>and</u> B occur, sometimes referred to as the <u>joint occurrence</u> of A and B (Ross, 1988).

Focusing now specifically on the X_i, recall that we have defined a marginal criterion measure as $F(x) = P\{\gamma : X(\gamma) < x\}$. We have verbally described a marginal criterion measure as the probability that the raw performance of the candidate being evaluated exceeds that of any other candidate system on the single criterion of interest. Alternately, we



$\Sigma X_i \leq 1.0$

Figure 27. Venn Diagram of Intersecting Criteria

have described it as the portion of the set of candidate systems which is dominated on the ith criterion by the candidate system being evaluated. Since the elementary event for the ith criterion was defined as $(\gamma_i : X_i < x_i)$, the intersection or joint occurrence must represent the event that <u>both</u> $(\gamma_i : X_i < x_i)$ <u>and</u> $(\gamma_j : X_j < x_j)$ occur. Consequently, the probability of the intersection of two X_i must be verbally described as <u>the probability that the raw performance of the candidate being evaluated exceeds that of any other candidate system on both (or all) of the intersecting criteria</u>. Alternately, it may be described as <u>the portion of the set of candidate systems which is dominated on both (all) of the intersecting criteria</u> (Zeleney, 1977: 150).

Graphically, we may add a dimension for each intersecting criterion (Figure 28). Any candidate systems falling within the doubly shaded region would be described as dominated on both (or all, in the case of more dimensions) of the intersecting criteria. The corresponding portion of the total set of candidate systems would be the correct numeric value of the intersection term of CF.

Since we have demonstrated that the so-called inter<u>action</u> terms of CF are really inter<u>section</u> terms, the question arises, "In the design context, what is inter<u>action</u> and how does it relate to CF?" Interaction is typically discussed within the context of regression or linear models and is defined (Kleinbaum, Kupper, and Muller, 1988) as "the condition where the relationship of interest is different at

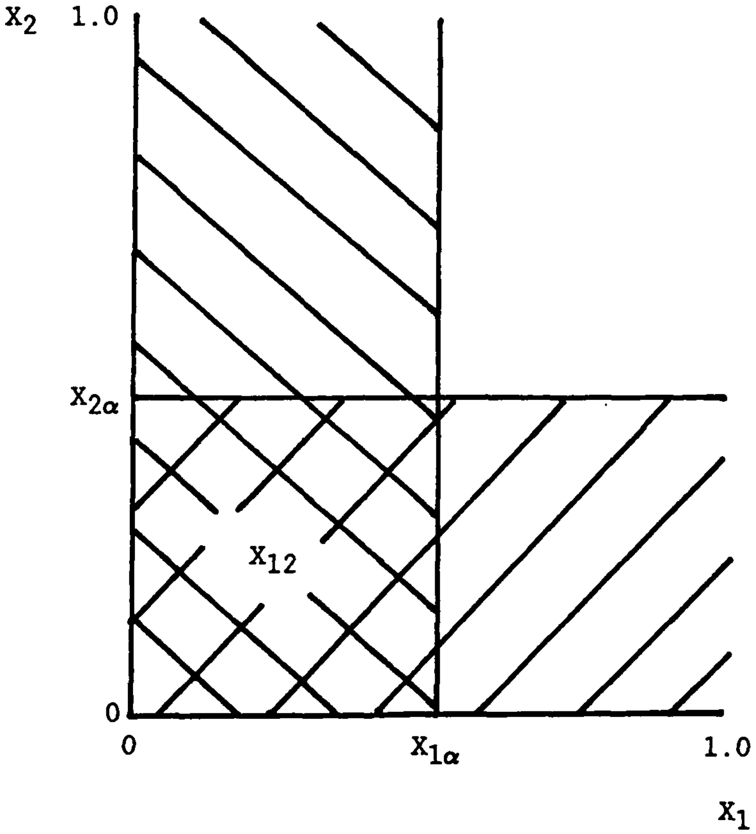


Figure 28. Dual Criteria Dominance

different levels (i.e., values) of the extraneous (control) variables". Conceptually in the design context, it is precisely what we have perceived it to be all along: a relationship which causes the value of one criterion to be "influenced by" (or "associated with") the value of some other criterion. As visualized in Figure 29, then, interaction is any relationship which would cause the candidate systems to be distributed throughout the X_1 space in a different manner, depending on the value of X_2. If the candidate systems were uniformly distributed throughout the X_1,X_2 space, then X_1 and X_2 would be described as independent and computation of X_{12} would become a simple matter of multiplying the intersecting marginal values (e.g. X_1*X_2). However, the presence of interaction precludes such simplicity.

For a moment, let us pause to consider the implications of the depiction in Figure 28. Such a figure could be constructed for the evaluation of each candidate system. Each candidate would have singly shaded areas corresponding to the marginal performance of that candidate on each criterion. These may be referred to as areas of partial dominance since they would contain candidates which are dominated on at least one criterion. The intersection of these shaded areas (the doubly shaded area) will cover the portion of the set of candidate systems completely dominated on the intersecting criteria. Conversely, any candidate systems falling in the <u>un</u>shaded area would completely dominate the candidate being evaluated. (At this point, we

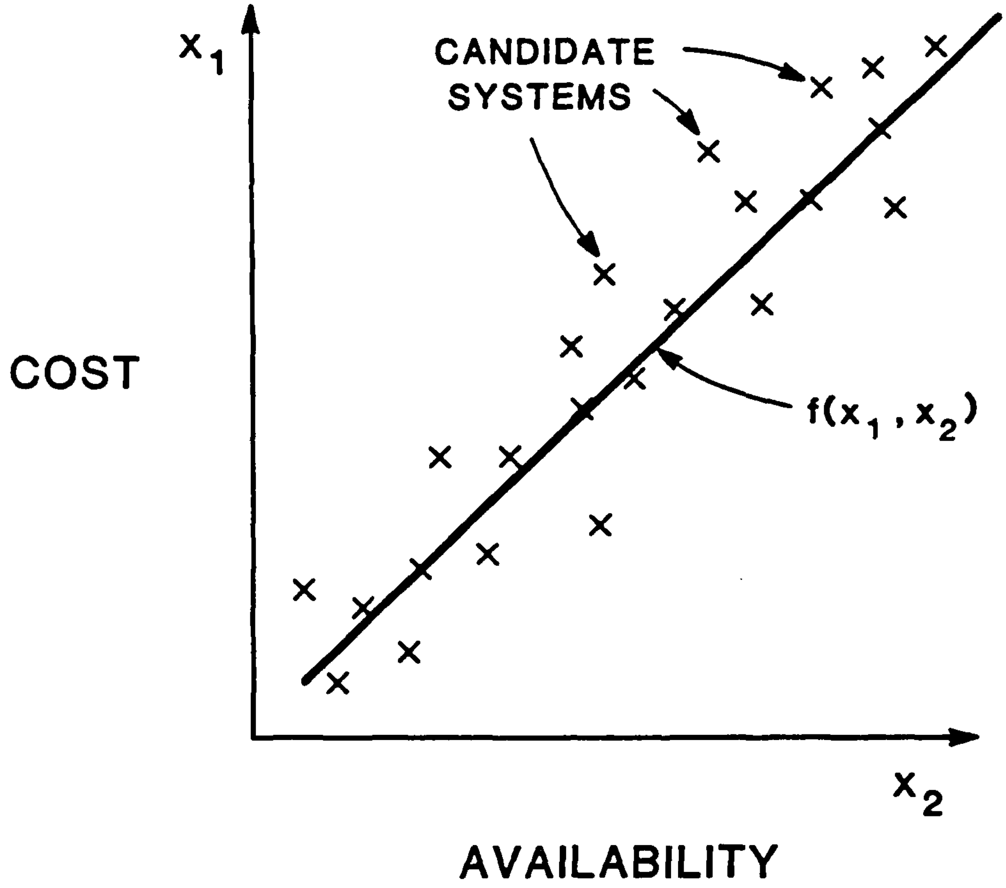


Figure 29. Functional Criteria Interaction

(Simons & Ostrofsky, 1988)

place no restrictions on the distribution of the set of candidate systems. Candidates may lie anywhere on the graph.)

The crux of the decision issue then becomes the ability to compare candidates characterized by different graphs in such a way that the "best" is chosen. Clearly, the criterion function attempts to achieve this by computing the probability associated with the covered area. Since the probability of the doubly-shaded area would be added twice by simply summing the marginal probabilities, it is subtracted once so that it is only counted a single time. This same approach is then easily extended to additional dimensions for the case of multiple ($n>2$) criteria, although visualization rapidly becomes difficult. But what is the logical rationale of the CF in summing areas of partial and total dominance? We shall defer the answer to that question until after additional developments.

Isolation of Inadequacy in the Current Method

Note that if F is a subset of E, then $P[F] \leq P[E]$ (Parzen, 1960: 20). Parzen has termed this relation "the inequality for the probability of a subevent." Since an intersection event is a subevent of its associated marginal events, the probability of any intersection must always be less than or equal to the probability of each associated marginal event. This is logical since an intersection requires the simultaneous occurrence of all relevant marginal events. For the simple case of two events, we state that

$$P(A\ B) \le \min\{P(A), P(B)\} \qquad \text{Eq. 4.30}$$

Sage (1981) refers to this simple relation as the conjunction rule. <u>It is precisely this relation which provides our first tangible clue as to a source of inconsistencies in past implementations of the criterion function approach</u>. Specifically, Peschke's (1986: 133) demonstration of Model V encountered X_{ij} which exceeded one of their marginal or lower order interaction criterion values (X_i, X_j, etc.) Although it is possible that these results were isolated anomalies, the following paragraphs substantiate the contrary.

The approach followed to compute interaction term values was described earlier in this chapter. Essentially, the approach consists of

1) characterizing the interaction between criteria as some function $g(x_1,x_2)$,

2) converting this function to a c.d.f. of the form $F(x_1,x_2|x_1)$, and then

3) using Bayes's rule to produce $F(x_1,x_2)$ by multiplying $F(x_1,x_2|x_1)$ by $F(x_1)$.

Ostrofsky (1968) first suggested this innovative approach as a means to the measurement of criteria interactions given the limitations of existing state of the art in statistical methods. He

perceived that the achievement of a two-dimensional projection of the multivariate probability density function might facilitate curve-fitting via the simple, yet effective Kolmogorov-Smirnov test. Unfortunately, the following evidence indicates inconsistencies in this method of computing interactions.

Note that both of the functions $F(x_1)$ and $F(x_1,x_2|x_1)$ are non-decreasing as x_1 increases, since this is a necessary characteristic of any cumulative distribution (Parzen, 1960: 173). Since $F(x_1,x_2)$ is computed as the product of these quantities, this means, in turn, that $F(x_1,x_2)$ is non-decreasing as x_1 increases. This result is undesirable since as $F(x_1)$ increases, $F(x_1,x_2)$ may easily exceed $F(x_2)$ for a given candidate system in violation of the conjunction rule. Such a situation is depicted in Figure 30. Note that, in the example, as $F(x_1)$ exceeds .5, the associated values of $F(x_2)$ would begin to decrease. Correspondingly, the area of intersection begins to decrease. However, using the current method of computing $F(x_1,x_2)$, we can see that as $F(x_1)$ approaches 1.0, $F(x_1,x_2|x_1)$ also approaches 1.0 and, therefore, so does their product, $F(x_1)*F(x_1,x_2|x_1)$, which is currently perceived to be $F(x_1,x_2)$. Consequently, we see that in this case, the current method causes $F(x_1,x_2)$ to continuously approach 1.0 even though $F(x_2)$ is approaching 0. Clearly, at some point, $F(x_1,x_2)$ will exceed $F(x_2)$. Therefore, we see that any function representing the intersection which is non-decreasing in x_1 is inconsistent with the conjunction rule.

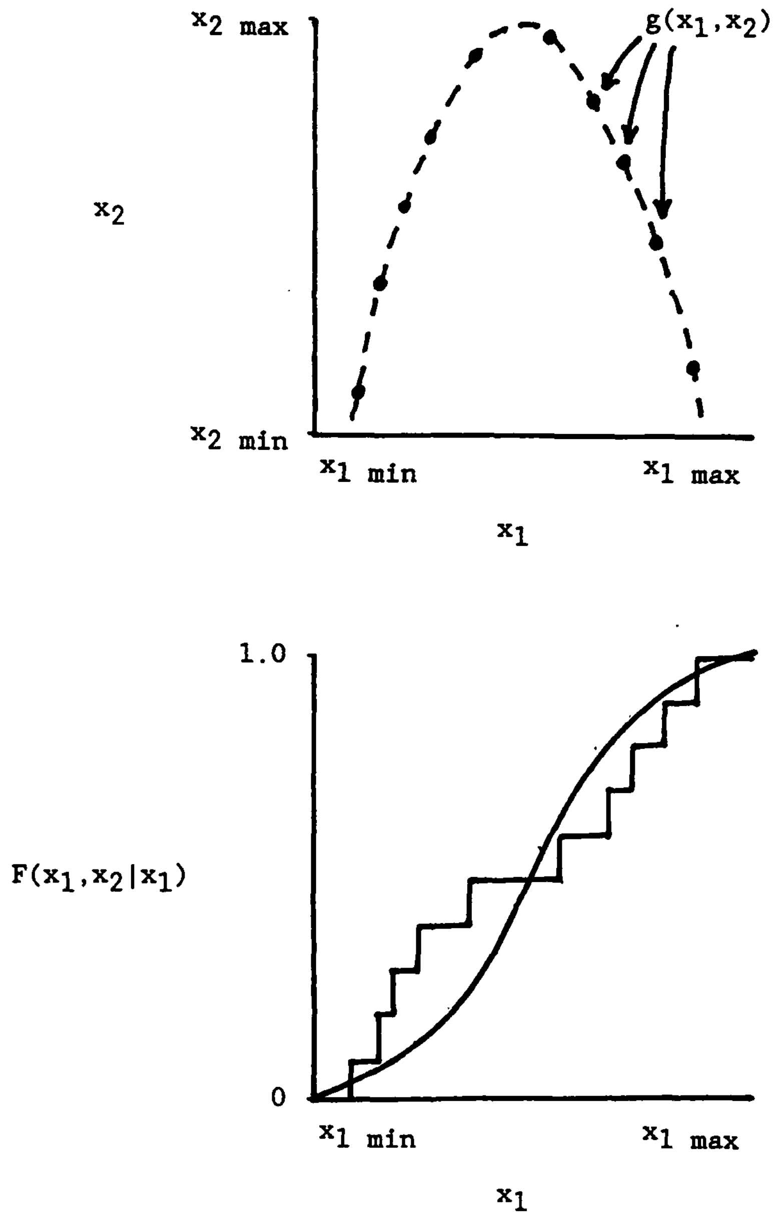


Figure 30. Conversion of Interacting Criteria

Furthermore, note that a c.d.f. is a cumulative representation of a density function. Note further that a univariate density function requires a two-dimensional visual representation (one dimension for observed values, one for frequencies). Similarly, a bivariate density function requires a three-dimensional visual representation (one dimension for each variable or criterion plus one for the frequencies). Consequently, the curve $g(x_1,x_2)$ in the two-dimensional Figure 30 does not contain <u>all</u> the information of the density function $f(x_1,x_2)$. Instead, $g(x_1,x_2)$ is a two-dimensional projection of the multivariate probability density function $f(x_1,x_2)$. The two dimensions represented are x_1 and x_2. The third (and missing) dimension is that of <u>frequency</u>. In order to represent a three dimensional function, what is required is a two-dimensional projection given the <u>third</u> dimension (i.e. at a given frequency), e.g. $AB|\underline{C}$. By contrast, the current method uses the same dimension (x_1) twice, e.g. $AB|\underline{A}$. Therefore, the dimension of relative frequency is lost. This accounts for the inappropriate nondecreasing behavior described above.

Measurement of Criteria Intersections

If the existing method of computing intersections and/or interactions is inadequate, what then is the correct method? First, it is important to reiterate that, as demonstrated previously, the entity required by the criterion function and represented by the term X_{ij} is, in fact, the inter<u>section</u> of X_i and X_j. The presence of inter<u>action</u> simply complicates the computation of inter<u>section</u> values.

As stated previously, the probability of an intersection of events is frequently called a joint probability distribution (Feller, 1966: 67). Since we seek to employ the cumulative distribution function as a measure of criteria performance, our goal is then to obtain the joint cumulative probability distribution function of the intersecting criteria.

Feller (1966) introduces multivariate distributions beginning with consideration of the case of a two-dimensional Cartesian plane. He states that for any region Ω, the density f attributes the probability

$$P\{\Omega\} = \iint_{\Omega} f(x_1, x_2)dx_1dx_2 \qquad \text{Eq. 4.31}$$

For a specific area bounded by the values a_1, b_1 in the x_1 dimension and a_2, b_2 in the x_2 dimension we have

$$P\{a_1 < x_1 \leq b_1, a_2 < x_2 \leq b_2\} = \int_{a_1}^{b_1}\int_{a_2}^{b_2} f(x_1, x_2)dx_1dx_2 \qquad \text{Eq. 4.32}$$

for all combinations where $a_i < b_i$. By letting $a_1 = a_2 = -\infty$ we obtain the desired distribution function F of f

$$F(x_1, x_2) = P\{X_1 \leq x_1, X_2 \leq x_2\} \qquad \text{Eq. 4.33}$$

In other words, the intersection is computed as the volume of the space under the density function $f(x_1,x_2)$ bounded by the values $x_1=b_1$ and $x_2=b_2$. It is precisely this relationship which determines the value of the intersection terms in the criterion function. This relationship is easily extended to the case of n criteria as

$$F(b_1,b_2,\ldots,b_n) = P\{X_1 \le b_1,\ X_2 \le b_2,\ \ldots\ ,\ X_n \le b_n\} \qquad \text{Eq. 4.34}$$

$$= \int_{-\infty}^{b_n}\int_{-\infty}^{b_{n-1}}\cdots\int_{-\infty}^{b_1} f(x_1,\ldots,x_n)\,dx_1\ldots dx_n \qquad \text{Eq. 4.35}$$

Note that this relationship was clearly recognized and described by Ostrofsky (1968, 1977) (See Appendix A of this dissertation, Equations A.43 and A.56.) Presumably, however, recognition of its significance became overshadowed in the effort to achieve the best possible means of representing a continuous multivariate density function to facilitate the measurement of interactions. However, as the preceding discussion reveals, the method based on two-dimensional projections is capable of producing results which are inconsistent with the conjunction rule. We are therefore forced to return to the best conventional means available for representing a density function in order to permit the use of Equation 4.35 for the computation of intersection terms.

When criteria are independent, the computation of intersection

values is greatly simplified. In this case, we have (Parzen, 1960: 295; Ross, 1988)

$$F(x_1,x_2) = P\{X_1 \le x_1,\ X_2 \le x_2\} \qquad \text{Eq. 4.36}$$

$$= P\{X_1 \le x_1\} * P\{X_2 \le x_2\} \qquad \text{Eq. 4.37}$$

$$= F_{X1}(x_1) * F_{X2}(x_2) \quad \text{for all } x_1, x_2 \qquad \text{Eq. 4.38}$$

More generally, for the case of n independent criteria

$$F(x_1,x_2,\ldots,x_n) = \prod_{i=1}^{n} P\{X_i \le x_i\} \qquad \text{Eq. 4.39}$$

$$= \prod_{i=1}^{n} F_i(x_i) \qquad \text{Eq. 4.40}$$

Visualized graphically in three dimensions, the intersection of two independent criteria might appear as in Figure 31. Since the criteria are independent, the density function is the same in the X_1 space, regardless of changes in X_2 (and vice versa). Recall now that when criteria inter<u>act</u>, the relationship of interest (the density function, $f(x_1,x_2)$) is different at different values of x_1 and x_2. Graphically, interaction might produce a density function of the sort shown in Figure 32. No longer can the intersection value $F(x_1,x_2)$ be computed simply as $F_1(x_1)*F_2(x_2)$. Instead, it will be necessary to formulate a continuous density function, $f(x_1,x_2)$ and compute X_{ij} using Equation 4.31 (or, more generally, Equation 4.35). Therefore, <u>while tl requirement for intersection terms remains the same, the significance of interaction is that it complicates (perhaps substantially) the computation of the intersection (X_{ij}) values</u>.

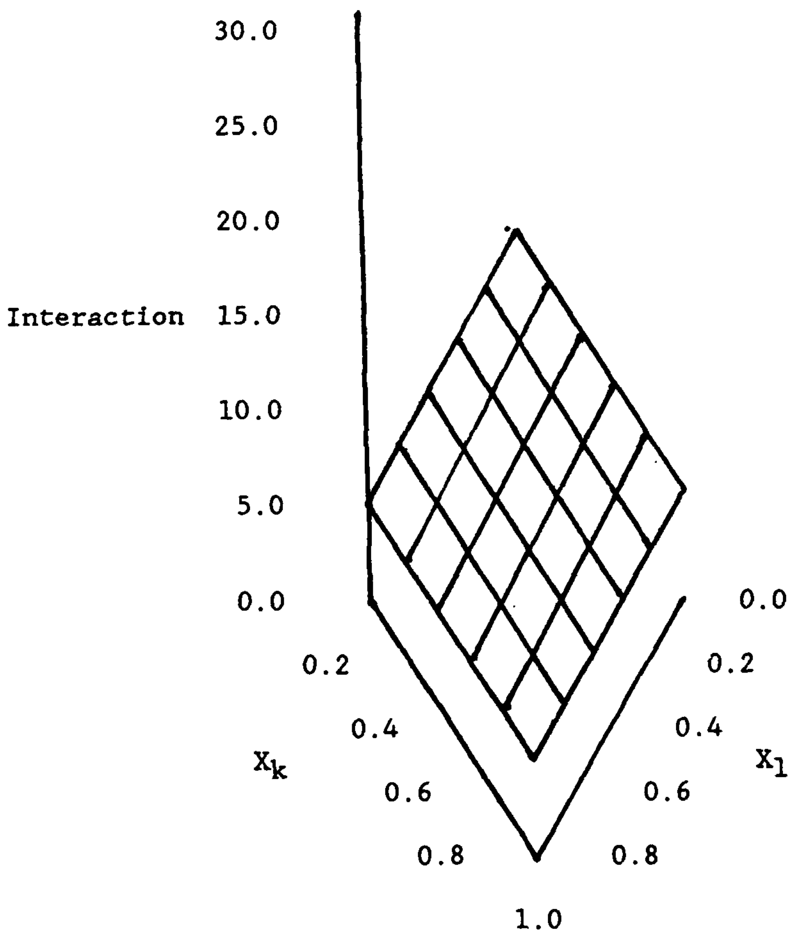


Figure 31. Independent Criteria Intersection

(Adapted from Folkeson, 1982)

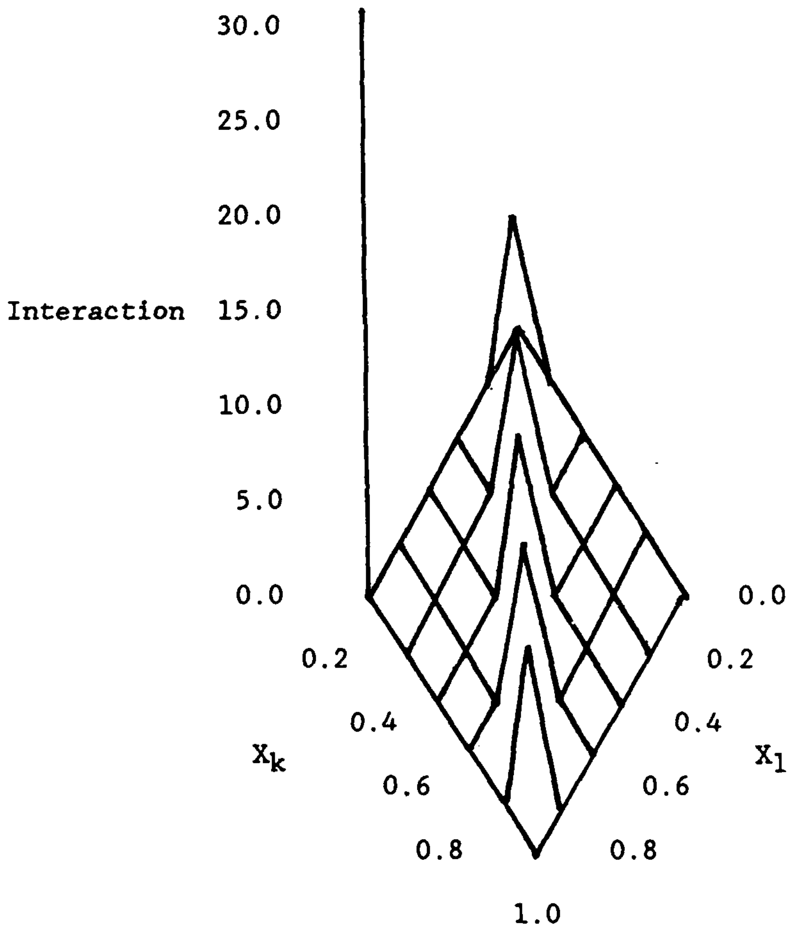


Figure 32. Interacting Criteria Intersection

(Adapted from Folkeson, 1982)

Note that in some cases, it may not be possible to fit a continuous density function even though the presence of interaction is too great to ignore. In such cases, we may choose to treat the distribution of candidate systems as discrete and accurately represented by the distribution of identified candidate systems. Given these assumptions, we could then compute intersections by simply counting the number of candidate systems dominated on all of the intersecting criteria and dividing by the total number of candidate systems to produce the desired proportion.

An important implication of this "rediscovery" (although tangential to this specific research), is the implicit assertion that even Models I-IV of the criterion function should include intersection terms. Models V-VIII would still be distinguished by the presence of interaction. However, the difference between the two classes of models would no longer lie in the presence or absence of intersection terms, but rather in the manner in which those terms are computed. In other words, Models I-IV have intersection terms while Models V-VIII have intersection terms with interaction. Therefore, the criterion function for all eight models would be computed using Equation 4.6 including intersection terms (i.e. all $\delta=1$). However, for Models I-IV (independent criteria), Equation 4.40 would be used to compute the X_{ij} intersection terms, while Models V-VIII (interacting criteria) would require intersection computation via Equation 4.35. Since the latter case may require the estimation of a continuous multivariate density

function, regression techniques and/or use of a Chi-Square or more robust tests may be required.

Further support of the need for (independent) intersection terms may be found in Parzen's (1960: 88) observation that "two mutually exclusive events, A and B, are independent if and only if P[A]P[B]=0, which is so if and only if either A or B has probability zero." In out context, this means that for two design criteria to be <u>both</u> mutually exclusive <u>and</u> independent, the probability associated with at least one of the criteria must be 0 when the other exists. Under our current assumptions, this should not occur. Therefore, either the criteria are not mutually exclusive <u>or</u> they are not independent. Since we accept the fact of their independence (this is the very basis for the selection of Models I-IV as opposed to Models V-VIII), we conclude that they must not be mutually exclusive (i.e. they intersect). Consequently, the computation of their union requires the subtraction of their intersection.

In reexamining the revised computation of intersection terms, it should be noted that using Equations 4.35 and 4.40, X_{ij} will always be less than or equal to the marginal criterion values, X_i and X_j. This is easily seen graphically in Figures 31 and 32. Since X_{ij} is effectively a subset of X_i, it can never be greater than X_i. Therefore, we have achieved compliance with the conjunction rule (Equation 4.30). What, however, has been the impact on the value of

CF itself and, more importantly, have we completely solved the observed discrepancies in CF values?

To examine the effect of intersection terms on CF, consider the trivial case of two independent criteria of equal relative importance. Now assume the existence of two candidate systems (A,B) whose criterion values are $X_1 = .1$, $X_2 = .9$ for A and $X_1 = .5$, $X_2 = .5$ for B. Initial inspection might infer that since the two criteria are equally important, these two candidate systems are equally preferable. To see this, we define isopreference lines to be lines which connect points of equal preference (a la Keeney and Raiffa, 1976). Then for two equally important independent criteria, we might expect isopreference lines to be a family of straight lines of the form $X_1 + X_2 = C$ where $0<C<2.0$. These lines would appear as in Figure 33 and, as suggested, candidates A and B would seem to fall on the same line, indicating equal preference.

Given that the marginal relative weights are equal, the sum of the marginal θ_i will be equal for the two candidate systems (due to the commutative property of multiplication) since the sum of the marginal X_i equals 1.0 in both cases. Now recall that since X_1 and X_2 are independent, the intersection terms, X_{12}, of A and B are computed as $.1*.9 = .09$ and $.5*.5 = .25$, respectively. Since the intersection term is subtracted in CF (Equation 4.6) and since the relative weights, a_1 and a_2, will be equal, we can see that for any positive value of a_{12}, candidate A will be preferred to candidate B because B

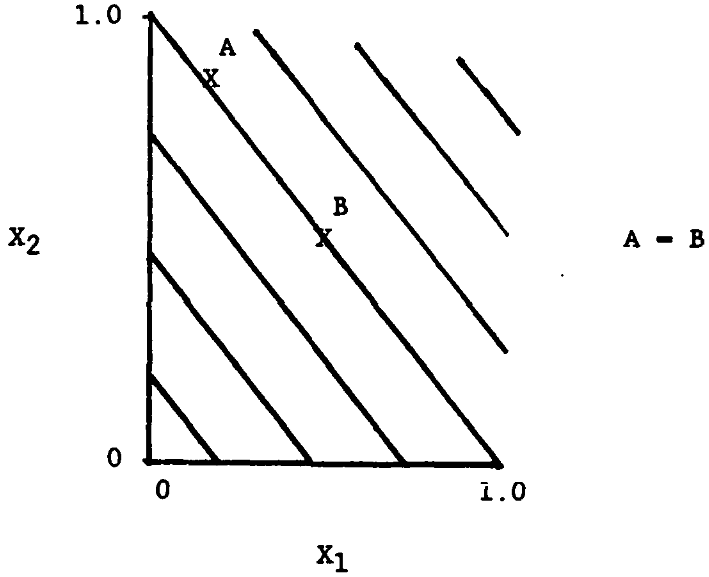


Figure 33. Isopreference Lines

has a larger intersection term. In terms of isopreference lines, subtracting interaction terms clearly makes the lines convex (e.g. Figure 34) where the degree of curvature would be determined by the magnitude of a_{12}, thus redefining the isopreference lines. Since an inverse relationship between the size of the intersection term and the value of CF may seem counterintuitive, we seek to understand the underlying (theoretical) rationale.

The reason for this behavior can be seen by examining the areas of dominance associated with the two candidate systems. Candidate A (Figure 35) has larger areas of partial dominance and a smaller area of complete dominance (intersection). By contrast, candidate B (Figure 36) has smaller areas of partial dominance and a larger area of complete of complete dominance. Intuitively, we might suspect that B's larger area of complete dominance would make it preferred.

However, recall that CF selects the candidate with the largest total area of the graph covered (via either partial <u>or</u> complete dominance). Recognize that as the covered portion of the graph gets larger, the <u>uncovered</u> portion gets smaller. Since the covered portion represent those candidates which are dominated by the one being evaluated on at least one criterion, the uncovered area represents undominated candidates, or, more to the point, <u>candidates which would dominate the one being evaluated</u>. Therefore, we can see that the evaluation accomplished by CF is best understood by examining the behavior of the uncovered rather than the covered area.

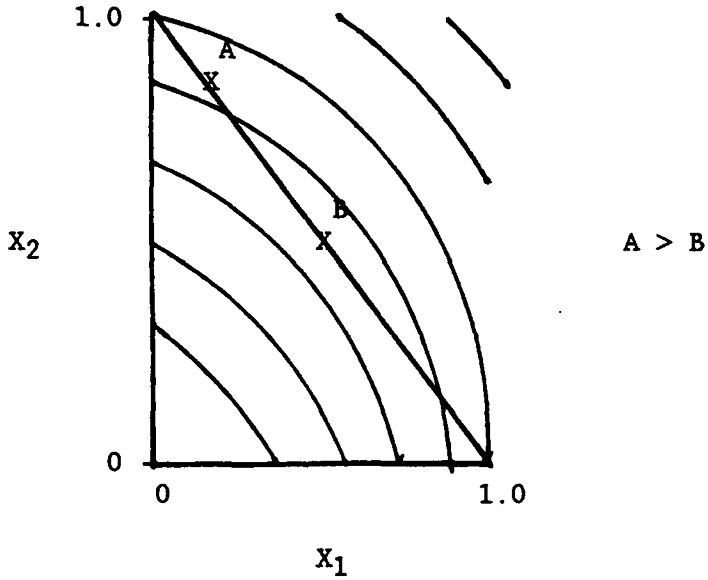


Figure 34. Isopreference Curves Caused by Subtraction of Intersections

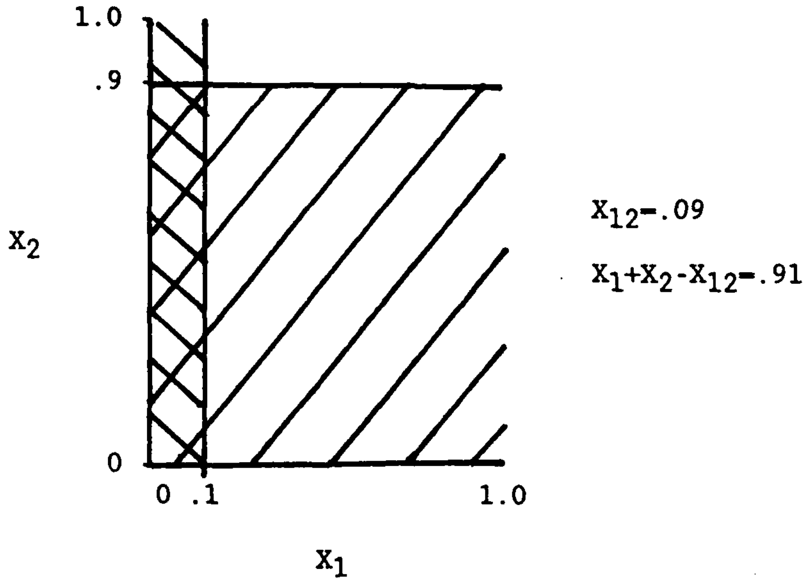


Figure 35. Areas of Dominance for Candidate A

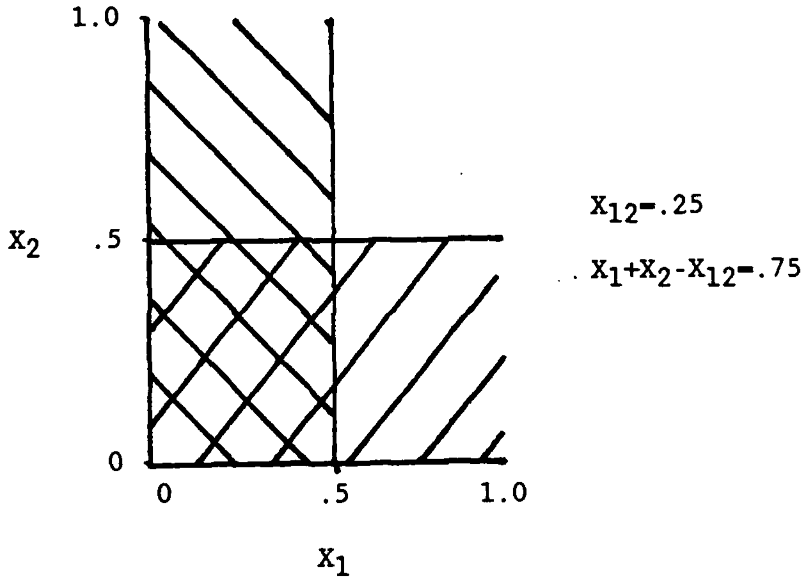


Figure 36. Areas of Dominance for Candidate B

Specifically, it can be seen that <u>CF favors the candidate system which minimizes the probability that it might be completely dominated by some other system</u> (1 - CF).

Observe how this understanding parallels the earlier, more general discussion of the rationale for the form of the criterion function in terms of θ_i. Specifically, we recognize that the behavior of the criterion function is consistent with the concept of attempting to minimize the probability of a bad choice. In other words, we seek the candidate system which is "farthest" from the worst possible choice. Therefore, candidates which are equally bad are indistinguishable. Figure 34 supports this interpretation, showing that (in the case of equal relative weights) points which are equidistant from the origin (the worst possible choice) share the same isopreference curve. (By contrast, decision making approaches which seek to get as close as possible to the theoretical ideal would reflect isopreference curves composed of points equidistant from the upper right-hand corner instead of the origin.)

With this understanding, the performance of CF with respect to our example candidates A and B becomes quite logical. Note that although B's area of total dominance is larger than A's, the increase is achieved at the expense of the uncovered area. In other words, candidate B has a greater probability that some other candidate exists which dominates B on all criteria. Simplistically, there is a greater

chance of finding a system which is indisputably superior to B than of finding one superior to A.

Although we can now logically explain the behavior of CF, can we be sure it will remain in the [0,1] range? Examination of Equation 4.6 (the criterion function with relative weights) reveals that we can make no such assertion without additional knowledge of the relationships among the marginal and intersection relative weights (a_i, a_{ij}, etc.). Although we now know that X_{ij} will always be less than or equal to the associated X_i, it is still possible that a_{ij} could be so large compared to the a_i that the overall value of CF becomes negative. Therefore, we examine the inclusion of relative weights in the next section.

Inclusion of Relative Weights

Intersection Relative Weights

Before proceeding, we acknowledge that the notation used so far for the representation of relative weights has been somewhat loosely specified. Prior to this point, we have attempted to remain consistent with the previous work being discussed. However, from this point on, we implement the following standard notation and recommend its continued use in future research:

Let

a_i = the relative weight of the ith criterion as originally specified by the decision maker

a_i' = the ith relative weight following "vertical" normalization

a_i'' = the ith relative weight following both "vertical" and "horizontal" normalization

A_i = the ith relative weight as finally included in the computation of CF

A second subscript may be used to denote the value for a specific candidate system and a superscript may be used to indicate the relative weight within a discrete interval of the X_i range. Note that under current practice, $A_i = a_i''$. However, the following section proposes changes in this regard.

Ostrofsky (1977d, 1987, 1988) has defined relative weights $\{a_i\}$ as subjective measures of the relative importance (usually subjective) of the decision criteria. As have Fishburn (1970) and Wymore (1976), he has also suggested (1977d) that relative weights may be treated as subjective probabilities. Consequently, current practice in implementing the criterion function (Ostrofsky, 1977d, 1987) requires that all relative weights sum to 1.0. Since intersection (heretofore "interaction") terms must be considered events in their own right (Loeve, 1963), it has been required that relative weights be assigned to them as well (e.g. Peschke, 1986) under the pervasive restriction that the sum of <u>all</u> relative weights equals 1.0. For example, in a three-criterion case, the relative weights in Table 10 would be

TABLE 10.

EXAMPLE RELATIVE WEIGHTS

Criteria	Relative Weights
X_1	.1
X_2	.05
X_3	.2
X_{12}	.1
X_{13}	.2
X_{23}	.2
X_{123}	.15
	1.00

considered an acceptable combination. These weights could be visualized in the form of a Venn diagram (Figure 37).

We have previously shown that, as probabilities, the θ_i and X_i are both required to satisfy the conjunction rule. Since the relative weights are probabilities in their own right, we know that the following conjunction relation must <u>also</u> be satisfied:

$$P(a_{ij}) \leq \min\{P(a_i), P(a_j)\} \qquad \text{Eq. 4.41}$$

Yet, as currently implemented, there is no assurance that this relationship will be satisfied (e.g. Peschke, 1986: 90). In order to avoid its violation, there might be a temptation to simply add a procedural constraint that Equation 4.41 must be satisfied in the initial definition of relative weights. However, as would have been the case with the functional interaction terms, such a stopgap effort would serve only to obscure an underlying source of inconsistency which requires revision.

Reexamination of the Venn diagram (Figure 37) reveals that, contrary to current practice, the marginal relative weights used in the computation of CF should not be as defined in Table 10. Rather, the complete marginal relative weight consists of the sum of the events which comprise it. In other words, <u>the relative weight of a marginal criterion (when placed into the criterion function) must include the relative weights of all intersections of which it is a</u>

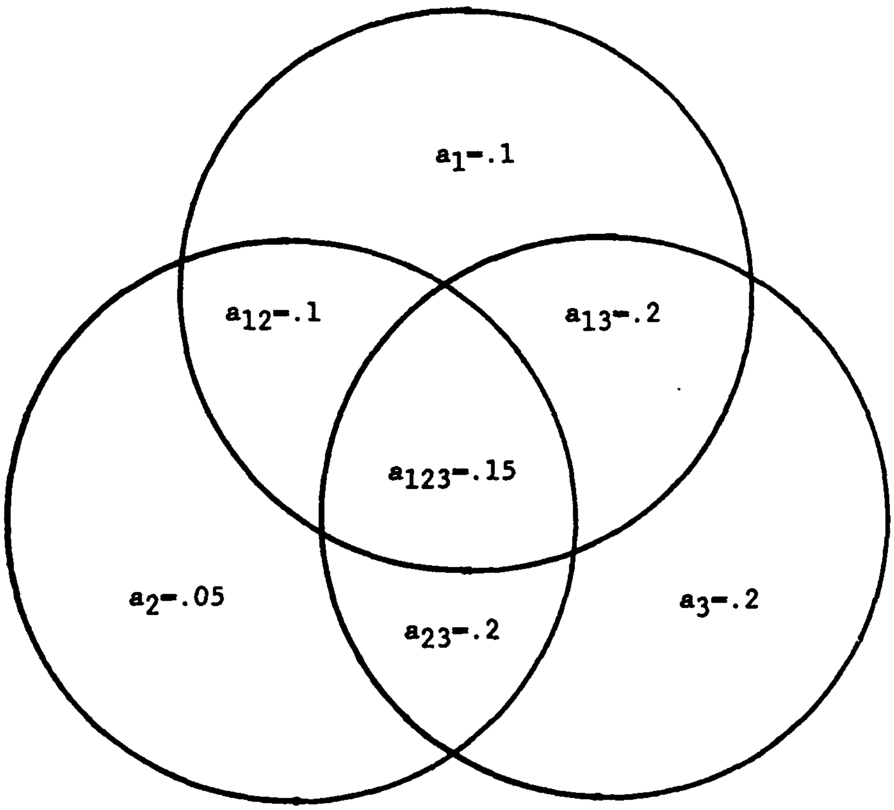


Figure 37. Venn Diagram of Relative Weights

<u>part</u>. Similarly, the relative weight of an intersection term must include the relative weights of all higher order intersection terms included within it. As a generic relationship, we state that

$$A_i = a''_i + \sum_{\substack{j=1 \\ i\neq j}}^{n} a''_{ij} + \sum_{\substack{j=1 \\ i\neq j \\ i\neq k \\ j\neq k}}^{n} \sum_{k=1}^{n} a''_{ijk} + \ldots + \sum_{\substack{j=1 \\ i\neq j \\ \cdot \\ \cdot \\ l-1\neq l}}^{n} \ldots \sum_{l=1}^{n} a''_{i\ldots l} \qquad \text{Eq. 4.42}$$

Note that this realization infers that the a_i'' as currently elicited and used do not include the a_{ij}''. (This fact is seen in Figure 37.) Therefore, the a_{ij}'' are not currently the Boolean intersections of the a_i'' and a_j''. However, with the additional operation of Eq. 4.42, the A_{ij} do, in fact, become the intersections of the A_i and A_j, satisfying the conjunction rule (Eq. 4.41).

This relationship, combined with the previous discussion concerning the X_{ij}, leads to substantially improved insight concerning the meaning of the a_{ij} as currently defined. Since the X_{ij} represent the measure of performance on criteria i and j simultaneously, the a_{ij} can be seen to represent the importance of a <u>balance</u> in performance among the intersecting criteria (possibly achieved at the expense of performance on marginal criteria values). Since, for any particular application, no specific relationship may be assumed a priori concerning the importance of marginal criteria vs. criteria balance, it is not appropriate to mandate any particular relationship between

the a_i and the a_{ij}, other than that they sum to 1.0 (as achieved via normalization). Therefore, the a_{ij} may be either greater than or less than the a_i. However, Equation 4.42 is required to reflect the hierarchical relationship among intersection terms which becomes necessary when relative weights are collectively placed in the context of the criterion function. In other words, Eq. 4.42 is the means by which the independently identified a_i and a_{ij} are made consistent with the laws of probability intersections and, therefore, with the X_i and X_{ij}.

Therefore, for the purposes of computing CF in the example problem, we would revise the relative weights as shown in Table 11. Note that this approach for handling a_i is consistent with the definition of the X's in which X_i and X_j each include X_{ij}.

As partial support for the validity of this proposed change, consider candidate system performance at both ends of the spectrum. It is expected that for the worst theoretically possible candidate (all X_i=0), CF should be 0 and for the theoretic ideal (all X_i=1), CF should equal 1. For the worst candidate, both the current and proposed methods would, as desired, yield a value of 0. However, for the best possible system, the current method may well produce some value less than 1, since all A_i sum to 1.0 yet some A_{ij} are subtracted rather than added. If the sum of the subtracted relative weights is greater than that of those added, CF may even become negative. By

TABLE 11.

REVISED RELATIVE WEIGHTS

Criteria	Relative Weights
X_1	.1+.1+.2+.15=.55
X_2	.05+.1+.2+.15=.5
X_3	.2+.2+.2+.15=.75
X_{12}	.1+.15=.25
X_{13}	.2+.15=.35
X_{23}	.2+.15=.35
X_{123}	.15

contrast, the proposed method would produce CF=1 as desired, regardless of the values of the relative weights.

The presence of the a_{ij} and their distinction from the a_i constitute one of the more powerful and unique characteristics of the criterion function approach to decision making. While most other approaches are based on some specific combination of marginal criteria performance, the criterion function permits explicit tradeoffs among not only marginal criteria, but also all possible criteria combinations.

Merging Criteria Measures and Relative Weights

Finally, it remains to consider the merger of the "objective" $\{X_i\}$ and "subjective" $\{A_i\}$ terms in the form of the CF. Ostrofsky (1968, 1977d) logically considers this merger in terms of the intersection of the two probability measures associated with each event. In the simplest case of constant relative weights, Ostrofsky (1977d) observed that the values of A_i and X_i are independent: knowledge of the value of one has no effect on the value of the other. Therefore, the intersection is simply formed as A_iX_i and the criterion function emerges as the union of these intersections.

Clearly, in the case of varying relative weights as we have defined them, the assumption of independence between the criteria values and their relative weights is no longer valid. Now, the value

of A_i is intentionally influenced by the value of X_i. Fortunately, the method suggested for defining relative weights produces $A_i = f(X_i)$. Therefore, the resulting weight is actually $A_i|X_i$ and not simply A_i. Since

$$P(A \cap B) = P(A)*P(B|A) \qquad \text{Eq. 4.43}$$

we can still multiply the two values directly as

$$P(a_i \cap x_i) = P(a_i|x_i)*P(x_i) \qquad \text{Eq. 4.44}$$

where $P(a_i|x_i)$ is simply the normalized value of the relative weight and $P(x_i)$ is X_i. So although the underlying rationale is different from the case of constant relative weights, the mechanics of merging the $\{A_i\}$ and $\{X_i\}$ remain essentially the same. For simplicity's sake, we choose to ignore the distinction henceforth, using the symbol A_i in lieu of the more accurate $(A_i|X_i)$.

Normalization of Continuously Variable Relative Weights

The vertical and horizontal relative weight normalization scheme of Ostrofsky (1987) is a truly subtle and remarkable mechanism, meriting further study beyond the scope of this research. Note that without explicitly defining the complete preference structure of the decision maker, normalizing the $\{a_i\}$ first vertically and then

horizontally provides a means of implicitly responding to the presence of preferential interaction across criteria.

For the case of relative weights which remain constant <u>within</u> intervals but vary <u>between</u> intervals (Models II and VI), vertical normalization was achieved prior to evaluation of any specific candidate systems. Specifically, the relative weights in each interval were normalized a priori. These normalized weights were then used to evaluate CF for each candidate. The extension to the case of continuously varying relative weights (Models III and VII) then becomes a simple matter of revising the sequence, but not the content, of the vertical normalization scheme. The revised algorithm for determining relative weights is as follows:

A) Determine the function, $a_i = f_i(X_i)$ for each i.

B) For each candidate system:

  1) For each i:

    a) Compute $a_i = f_i(X_i)$

    b) Compute $a_j = f_j(X_i)$ for each j at that value of i

    c) Compute the vertically normalized value of a_i as

$$a_i' = a_i / \sum_j a_j \qquad \text{Eq. 4.45}$$

  2) Repeat step 1 for each criterion.

  3) Horizontally normalize the a_i for each candidate system to produce $\{a_i''\}$.

4) Revise relative weights to include related higher order intersection relative weights using Equation 4.42, producing $\{A_i\}$.

C) If more candidate systems remain, return to step B.

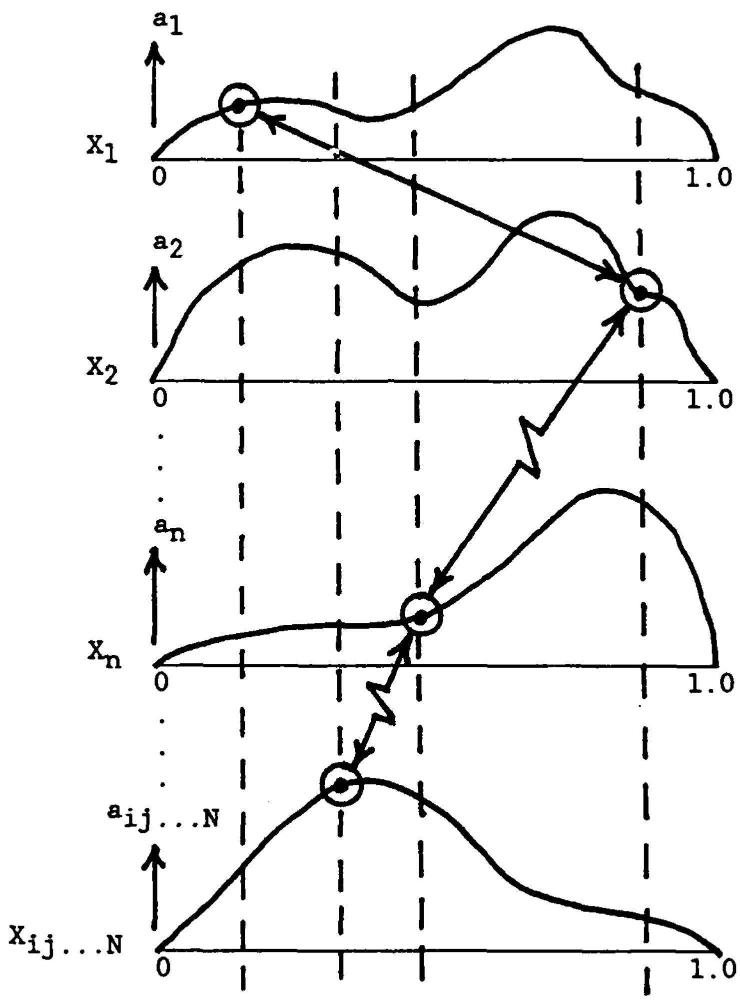
In sum, the revision simply defers vertical normalization until a specific value within the X_i range is identified for a given combination of candidate system and criterion. The obvious rationale for this revision in sequence is that it would be impossible to accomplish vertical normalization a priori with continuously varying relative weights since there are, effectively, an infinite number of X_i intervals in which a_i would have to be computed. Consequently, the determination of which intervals (points) should be normalized is deferred until it is known which points are <u>needed</u>. An example of the revised method is shown in Figure 38.

Empirical vs. Continuous Distribution Functions

Throughout this chapter we have generally assumed that the cumulative distribution function used to produce the X_i from the x_i will take the form of a continuous (probably nonlinear) function. This section discusses those cases in which we are unable or unwilling to use such a function.

The identification of a continuous c.d.f. would require the definition of a continuous function which "reasonably" approximates

Candidate System, α



Compute: $a_i = f_i(X_{i\alpha})$

$$a_{i\alpha}' = \frac{a_{i\alpha}}{\sum_{j=1}^{N} a_j}$$

$$a_{i\alpha}'' = \frac{a_{i\alpha}'}{\sum_{j=1}^{N} a_{j\alpha}'}$$

$$A_i = a_{i\alpha}'' + a_{ij\alpha}'' + \ldots + a_{ij..N\alpha}''$$

Figure 38. Normalizing Continuously Variable Relative Weights

the empirical data points observed. The enforcement of the "reasonableness" characteristic could be achieved through the use of an established statistical test, e.g. Kolmogorov-Smirnov, Chi-Square, etc. However, it is also possible (particularly in the case of complex interaction) that no function can be identified which is able to "pass" such objective tests. When this condition occurs, the decision maker faces at least three options.

One option is to assume independence among criteria. This option may be preferred in cases where visual partial plots of the data points present no consistent appearance of meaningful interaction. (This was the case with respect to functional interaction among the data used by Folkeson (1982).) Although it may not be technically accurate to assume that the criteria are truly independent in such a case, the premise is that if we are unable to clearly define or make sense of the nature of the interaction, we may be safer in ignoring it. An analogy might be that simple, naive forecasting models frequently perform better than more sophisticated models which assume the wrong underlying model (Makridakis, et al, 1982).

Another option is to simply fit the best continuous function possible and use it. The essence of this approach is simply doing the best one can with real world data. Curve-fitting techniques (i.e. regression) could be used with the intent of minimizing or maximizing some model selection criteria, e.g. R^2, $S^2_{y|x}$, F statistic, MSE, Mallow's C_p, PRESS, etc. (Kleinbaum, Kupper, and Muller, 1988). By

fitting a variety of models for different combinations of criteria <u>and by considering the intuitive nature of the potential interactions</u>, the decision maker may come to some conclusion concerning which interactions merit modeling, even at the risk of inaccuracy.

A third alternative is to use the empirical distribution rather than a continuous approximation. The remaining paragraphs of this section address this alternative.

Before proceeding with a discussion of the use of empirical distributions, we suggest that a strong case can be made for the use of empirical distributions even when approximation with a continuous function is feasible. First, recognize that applications of the criterion function could be divided into two general cases: those instances where the defined set of candidate systems is highly speculative or known to be incomplete and those instances where the candidate systems are well defined or firm and we do not wish to consider any additional alternatives. In the former case, we effectively assume an infinite number of candidate systems. One of the primary advantages of the continuous c.d.f. in this case is that its use in the search of the design space may lead to the discovery of previously unrecognized, and potentially superior alternatives. Therefore, use of a continuous c.d.f. may be highly beneficial. However, in the latter case, we are not interested in the discovery of new candidates. Instead, our focus is solely on the determination of the relative best from among those alternatives already specified. In

such a case, our real problem is representing the empirical data as accurately as possible. In such a case, the use of a continuous approximation may unnecessarily sacrifice accuracy. Therefore, use of the empirical distribution may be the most appropriate course of action. (In fact, in such a case one could well argue that no large minimum number of empirical data points must be available since no statistical inference or estimation will be attempted. Instead, probability is serving only as an objective guide to the evaluation effort.)

Feller (1966: 36) defines the empirical distribution function F_n of n points $a_1, a_2, \ldots, a_n$ as the step function with jumps of 1/n (on the y-axis) at points $a_1, a_2, \ldots, a_n$ (on the x-axis). Given this definition and the fact that the sample description space is now considered finite, the computation of the probability of an event is quite straightforward. For any event E on the sample space S, we have that (Parzen, 1960: 25)

$$P[E] = N[E]/N[S] = (\text{size of } E)/(\text{size of } S) \qquad \text{Eq. 4.46}$$

In other words, use of the empirical distribution function permits us to simply count the number of events meeting the desired description and compute that number's ratio over the complete set. For a particular criterion and candidate system, then, we would count the number of candidate systems whose performance (x_i) was equal to or worse than the performance of the candidate we are evaluating. We

would then divide the resulting number by the total number of candidate systems to obtain the X_i for the candidate we're evaluating. Similarly, for intersection values (x_{ij}), we need only count the number of candidates whose performance is exceeded by the candidate we're evaluating on <u>both</u> (or all) of the intersecting criteria and, again, divide by the total number of candidate systems.

Note the extreme simplicity of using the empirical distribution compared to the continuous function. First, we need not concern ourselves with fitting or evaluating curves (continuous functions). Second, we need no longer determine the values of $z_{j\ min}$, $z_{j\ max}$, $x_{i\ min}$, or $x_{i\ max}$. Third, the use of empirical distributions is made no more difficult by the presence of interaction. Finally, once the computations of the x_i values are complete, we are primarily only performing counting operations rather than computations. Therefore, the evaluation of candidate systems may well lend itself to direct spreadsheet quantification rather than requiring formal programming efforts, making it potentially more appealing to practitioners.

In closing, it should be noted that the intent of this section has not been to argue that empirical distribution functions should arbitrarily be used as opposed to continuous functions. On the contrary, we reiterate that whichever form is most appropriate for the application should be used. However, since historically, discussions of the criterion function approach have focused on the continuous case, this section was intended to identify circumstances under which

use of the empirical alternative might be appropriate and the benefits which might be expected to accrue. As always, these benefits must be weighed against the corresponding sacrifices.

Summary of Revisions

In the second half of this chapter we have clarified procedural methods of past and proposed implementations of the criterion function approach to design decision making. In the process, we identified current sources of inconsistency and identified two significant changes in the methods of implementing the relevant theory to resolve these inconsistencies.

First, we have shown that the criterion function requires the measurement of criteria inter<u>sections</u> and have distinguished them from the concept of inter<u>action</u>. We have shown that, when continuous approximations of distribution functions are used, the current method of computing the intersections (X_{ij}) must be replaced with either Eq. 4.35 or Eq. 4.40, depending on whether or not interaction is present. We subsequently described circumstances under which the use of empirical distribution functions may be appropriate, in which case the computation of intersections may be accomplished via counting of the appropriate intersection events and the use of Eq. 4.46.

Second, we have used similar reasoning to show that an additional step (Eq. 4.42) is required to include higher-order intersection

relative weights in the determination of A_i, satisfying probability theory in general and the conjunction rule in particular. As part of this discussion, we achieved greater insight to the meaning and significance of intersection relative weights, as well as the flexibility of their relationship to marginal relative weights. We have also described the appropriate method for accommodating continuously variable relative weights.

By establishing consistency with probability theory, the proposed revisions complete the ability of the criterion function to achieve the stated research objective. In the next chapter, we demonstrate the theoretical contributions provided here in the context of an actual design decision problem.

Chapter 5

A DESIGN DEMONSTRATION: SPACE SHUTTLE AUXILIARY POWER UNITS

Purpose of the Demonstration

The objectives of this dissertation were specified at the ends of Chapters 1 and 3. The primary focus of the research concerns the theoretical approach of the criterion function and the appropriate methods for applying that theory. These issues were addressed and resolved in Chapter 4. However, to enhance clarity and to show that the proposals of Chapter 4 produce the desired results, this chapter demonstrates the application of the revised criterion function implementation method to an actual complex design decision problem: the choice of a design for auxiliary power units for NASA's space shuttle.

It is appropriate to emphasize that the purpose of this chapter is to demonstrate the proposals presented in Chapter 4, not to actually solve NASA's problem since the data comes from the Johnson Space Center. While the former could be accomplished with the resources and information currently available, the latter would

necessitate a far more extensive effort, including a variety of spin-off studies and laboratory testing efforts which could be made possible only with dedicated funding and manpower allocation by NASA and its contractors. Consequently, the data used, the models constructed, and the results obtained should be accepted as useful for the demonstration purposes intended and for the general insight provided, but should not be misconstrued as representing the definitive solution to be pursued. It is hoped, however, that in addition to achieving the research purposes, the demonstration will provide a useful framework and first step toward the ultimate solution should NASA proceed with their current interest in this problem.

As a final note, it is recognized that an unfortunate aspect of the demonstration used is that it may unintentionally perpetuate the common misconception that system design methods are applicable only to hardware systems. Therefore, the reader is encouraged to observe the generic nature of the steps taken and to recognize their applicability to the design of any defined system. Conversely, the strengths of this particular demonstration are its realism, importance, complexity, cost, and visibility.

Hogarth and Makridakis (1981b) have concluded that "values and goals are difficult to communicate and often must be sensed by planners rather than directly obtained from policymakers." For this reason, construction of this demonstration was primarily achieved through repeated iteration of the following algorithm:

1) Exploratory discussions were held between the researcher and NASA personnel.

2) The researcher synthesized information gained from the discussions and written material presented by NASA into a structured representation of the decision problem.

3) The output of step 2 was then used as a "straw man" during follow-on meetings with NASA personnel. Required adjustments were made from this starting point.

It was felt by all involved that this approach led to a much more complete and expedient result.

Assumptions

Since the focus of the demonstration concerns the implementation of the criterion function itself, it is appropriate to point out key assumptions. In general, we recognize that activities prior and subsequent to the use of the criterion function fall largely outside the scope of this research and are, therefore, assumed to be accomplished completely and correctly. In particular, we make the following specific assumptions (Ostrofsky, 1977d: 331) which are consistent with previous research (Folkeson, 1982; Peschke, 1986):

1. A comprehensive knowledge of the candidate systems exists to the extent that the resulting criteria measures can be adequately

defined in terms of the pertinent design.

2. Those who are evaluating criterion performance and its relative value are experts to the extent that their decisions are rational, representing the maximum level of accuracy and knowledge at the respective stage of development.

3. The decisions formulated from the resulting criterion function consider only the criteria identified for the design of the system.

4. One candidate system does not dominate all others for all criteria so that a logical choice of the optimal system from the set of candidate systems is not apparent without further analysis.

5. The worth of the emerging, optimal system merits the effort involved in its choice.

Background

Subsequent to the first manned lunar landing in 1969, a presidential task group recommended that the United States develop new technological systems for space operation with initial activities directed toward the achievement of a new space transportation capability. Toward that end, in 1970 the National Aeronautics and Space Administration (NASA) initiated engineering, design, and cost studies leading to the development of the Space Transportation System (STS), the prime component of which would be the space shuttle. The final design, determined in 1972, called for a space shuttle vehicle (orbiter) with an external fuel tank, augmented by two reusable

external solid rocket boosters or SRB's (see Figure 39). (Note: The facts and figures cited in the remainder of this section were primarily obtained from NASA's Space Shuttle News Reference, 1989.)

The first reusable manned space flight system, the shuttle was designed for the purpose of delivering and retrieving a wide variety of payloads (e.g. satellites, experimental platforms, interplanetary vehicles, etc.) to and from earth orbit. The orbiter is approximately the same size as a DC-9 aircraft, measuring 121 feet long, 57 feet high, and 79 feet wide. It weighs approximately 150,000 pounds without fuel or its maximum payload of approximately 65,000 pounds.

A typical shuttle mission profile calls for a maximum crew of seven astronauts and mission specialists to launch from John F. Kennedy Space Center in Florida or Vandenberg Air Force Base, California. Launch is accomplished using thrust from both the orbiter's main engines and the SRB's. The external fuel tank and the SRB's are discarded during ascent. Following approximately 7 to 30 days of operations in orbit, the crew flies the orbiter back into the earth's atmosphere, landing at Kennedy, Vandenberg, or Edwards Air Force Base, California.

Currently, the orbiter uses hydraulic power to activate all flight control surfaces, landing gear, brakes, nose wheel steering, and engine controls. To provide the necessary hydraulic power, the orbiter employs three mechanical power generators which are referred

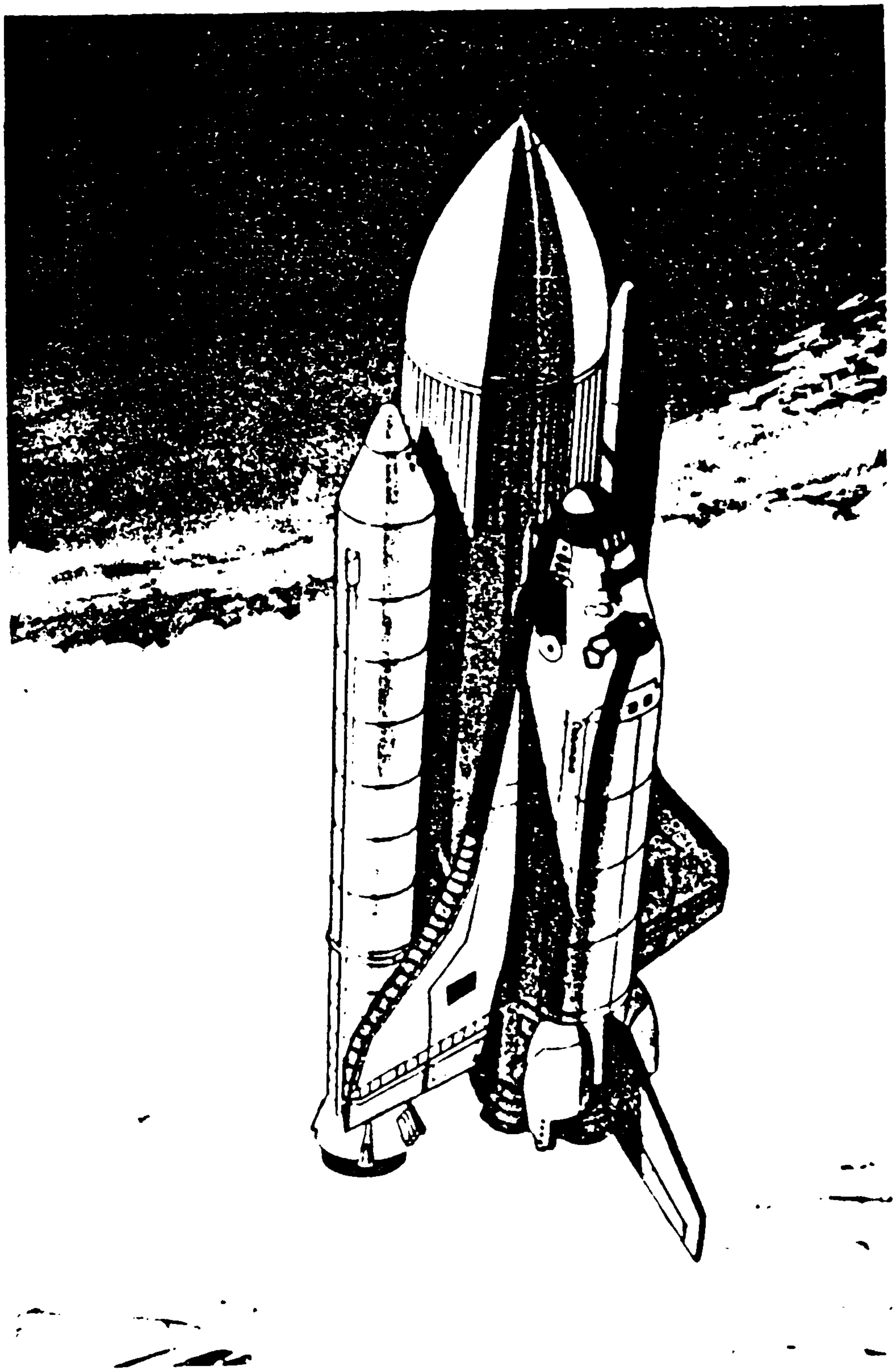


Figure 39. Space Shuttle System

(NASA Space Shuttle News Reference, 1989)

to as Auxiliary Power Units (APU's). The APU's (Figure 40) are 138-horsepower turbine engines weighing 85 pounds each which convert chemical energy from liquid hydrazine fuel into mechanical shaft power. The resulting mechanical power is converted to hydraulic power via variable-displacement, constant pressure hydraulic pumps. In addition to the engines, fuel cells, and hydraulic pumps, the APU subsystem includes a variety of supporting hardware such as fuel pumps, valves, and a water-spray heat exchanger cooling system. The total APU system weighs a combined 1860 pounds and occupies approximately 50 cubic feet of space at the rear of the orbiter payload bay.

The APU's are started several minutes before launch and are shut down approximately when orbit is achieved. All three APU's are required for this ascent phase. The APU's are reactivated for the descent and landing phases; however, operation of any two of the three APU's is considered sufficient to accomplish these activities.

Beginning with the initial design in the early '70's, NASA and Rockwell (the prime space shuttle contractor) managers and engineers considered the possibility of alternate APU designs. Concerns with the design selected and currently implemented included high weight, excessive servicing requirements, and safety hazards resulting from the use of hydrazine fuel. Renewed interest in alternate APU designs emerged in the early '80's with the advent of shuttle operations (first flight in April, 1981) and a few potentially serious

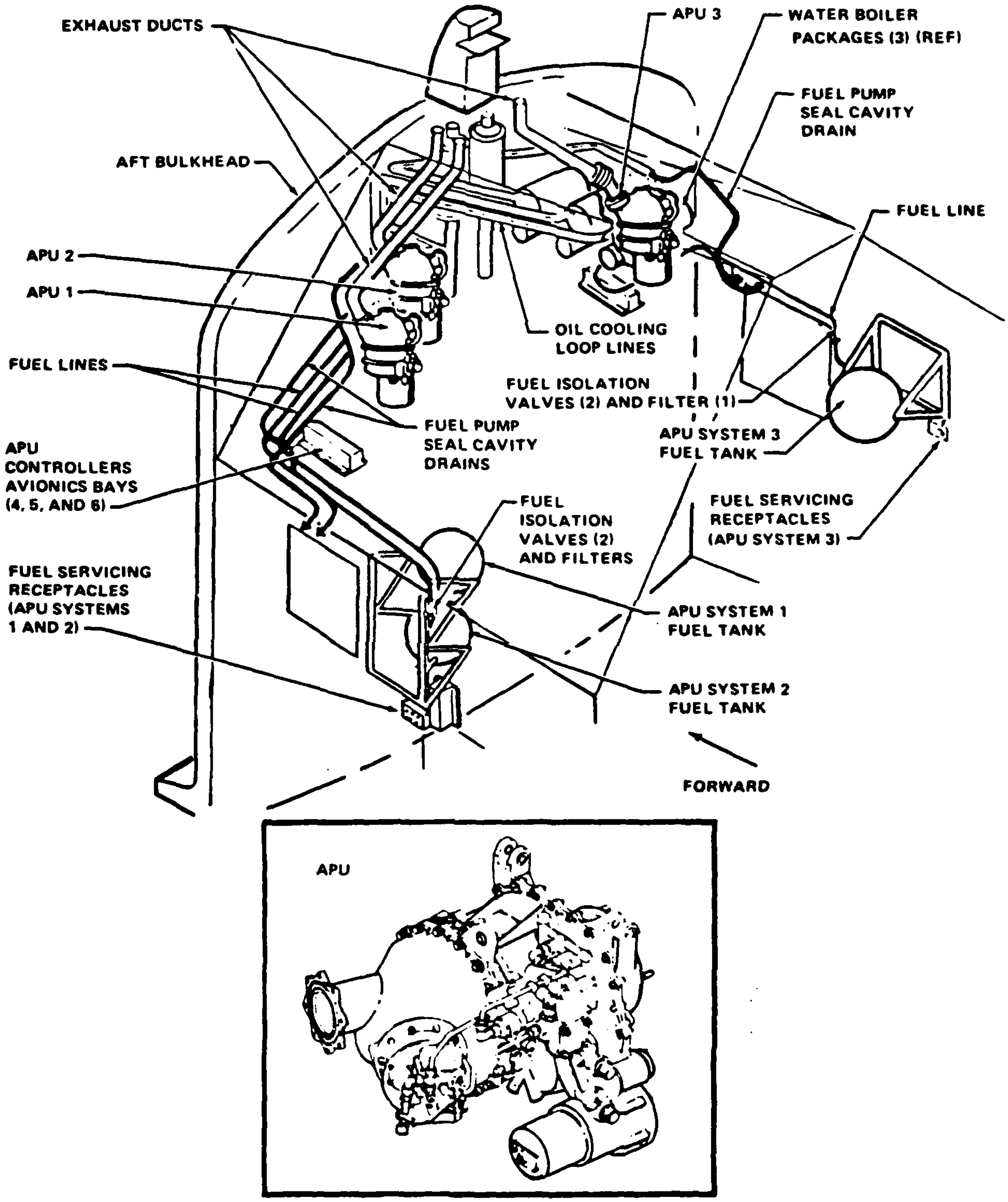


Figure 40. Auxiliary Power Units

(NASA Space Shuttle News Reference, 1989)

malfunctions during the first nine flights (1981-1983). At the time of this research, increased interest had once again become apparent as evidenced by a wide variety of "in-house" (NASA) and contractor studies underway. In addition to resolution of the concerns previously identified, a significant motive for reconsideration of APU design is the facilitation of electro-mechanical actuation (EMA) as an alternative to hydraulic power. Essentially, the use of EMA would involve replacing hydraulic actuation components with electric motors. This approach is appealing for variety of reasons, the foremost of which are weight reduction and substantial improvements in the efficient use of energy. (EMA systems could theoretically result in a net energy use of near zero since energy imparted to move control surfaces is returned to the system when the surfaces return to neutral positions.)

Feasibility Study

Needs Analysis

At the outset of data gathering for this demonstration, it was perceived that the problem to be solved concerned the broad issue of EMA for the space shuttle. However, subsequent discussions led to the realization that a related, but essentially independent first decision was required to address the selection of a design for a new APU to facilitate EMA. While the use of EMA was being considered primarily as a single general approach, the new APU design (typically referred

to as electric APU or EAPU) was known to require selection from among a variety of specifically identified known or theorized component designs, thereby readily lending itself to the type of analysis accommodated by the criterion function. Therefore, the EAPU was selected as the subject for this demonstration as a first step toward EMA.

Consequently, the "system" being designed is referred to as the Auxiliary Power Unit (APU). As described earlier, the APU system includes mechanical power generators which are used to drive hydraulic pumps which, in turn, provide hydraulic power for the following space shuttle flight control and utility systems:

- elevons
- rudder/speedbrake
- body flap
- SSME ATVC (space shuttle main engine active thrust vector control)
- gear deploy/uplock
- nose wheel steering
- brakes
- ME (main engine) control
- umbilical retract

Briefings and discussions with NASA managers and engineers produced the following broad statement of needs:

Provide a means for space shuttle APU power generation which provides low cost, high availability, and high safety

while providing performance equal to or greater than that of the current system.

Identification and Formulation of the Problem

As described in Chapter 2, this step involves specification of desired outputs, undesired outputs, environmental inputs, and intended inputs for each of the Production/Consumption Phases of the system life cycle. The following definitions were assigned to the APU life cycle phases:

Production Phase: All activities involved in the detailed design, development, planning, documentation, production, and procurement of the APU system and its planned integration into the space shuttle system.

Deployment (Distribution) Phase: All activities concerned with the integration of the APU system into the space shuttle system, to include hardware/software modifications, revision of space shuttle documentation, personnel training, and resupply (spares).

Operations Phase: The use and support of the APU system as an integral part of space shuttle operations.

Retirement Phase: The removal from operations or major modification of the APU system, whether concurrently with or independently from the retirement of the space shuttle system itself.

Under NASA's approach to system life cycle management, the Production Phase and portions of the Deployment Phase would be considered part of DDT&E (i.e. Design, Development, Test & Evaluation). The remainder of the Deployment Phase, as well as the Operations and Retirement Phases would be governed via Engineering Orders. It was also noted that NASA training, support, and spares considerations are typically handled through separate procurement

efforts as "spin-offs" of the primary design activities. Such efforts are initiated based on any perceived impact on existing or planned training, resupply, or datalink download (communication of in-flight data) requirements.

The input-output matrices derived for this demonstration are shown in Tables 12-15. The recurring themes reflected in the tables clearly follow the stated need: availability (including reliability and maintainability), safety, performance (to include low weight), and cost.

Synthesis of Solutions

While various specific design alternatives had been considered or discussed by NASA and Rockwell personnel, no evidence was found of an attempt to date to exhaustively define or evaluate a set of candidate systems. Therefore, the method described by Ostrofsky (1977d), which may be described as the strategy generation table (Howard, 1988), was employed.

A simplified representation of the current APU system is shown in Figure 41. One alternative concept is based on the use of batteries as an energy source in place of the current hydrazine fuel (Figure 42). Note that the use of batteries increases cooling requirements, quite possibly requiring the inclusion of a new, dedicated cooling system. A second new concept is based on the use of H_2/O_2 high energy

TABLE 12.

PRODUCTION PHASE

Desired outputs:

- Reliable components
- Maintainable components
- Safe components
- Compatible components
- Quality design, materials, and fabrication
- Low weight system

Undesired outputs:

- Expensive components
- Long lead times
- Unique or vulnerable material, production requirements
- High risk

Environmental inputs:

- Specifications for existing space shuttle systems, interfaces
- Contractor design, production capabilities
- Existing R&D results
- Off-the-shelf components

Intended inputs:

- New R&D
- Funding

TABLE 13.

DEPLOYMENT PHASE

Desired outputs:

- Efficient and timely integration with existing/planned space shuttle systems

Undesired outputs:

- Adverse impact on shuttle operations

Environmental inputs:

- Scheduled overhaul, "down" time (modification period)
- Existing contractor/NASA facilities, tools, equipment

Intended inputs:

- Additional/new facilities, tools, equipment
- Funding
- Qualified engineers, technicians, etc. to implement changes

TABLE 14.

OPERATIONS PHASE

Desired outputs:

- Ease of checkout, accessibility
- High operational availability
- High level of safety to operators and maintainers
- Low cost of operations
- Valuable information for future applications
- Minimum change to existing operations (transparency)
- Visibility of system performance
- Fail-op/fail-safe
- Good performance
- Low weight

Undesired outputs:

- Hazardous material utilization
- Unique tools, equipment
- High impact of system failures

Environmental inputs:

- Existing space shuttle hardware/software and interfaces
- Existing support capabilities and infrastructure (people, facilities, training, tools, etc.)

Intended inputs:

- Revised operations/support specifications
- Training for operators, maintainers
- Spares
- Funding
- Availability of consumables

TABLE 15.

RETIREMENT PHASE

Desired outputs:

- Inexpensive deactivation
- Maximum reusability of materials, personnel, equipment
- Ease of overhaul/rebuild
- Retirement consistent with spares program
- Inputs to product re-design, improvement
- Efficient disposition of retired hardware
- Ease of change implementation

Undesired outputs:

- Special handling/storage requirements
- Obsolescence

Environmental inputs:

- NASA plans for follow-on systems, operations

Intended inputs:

- Retirement plans
- Funding

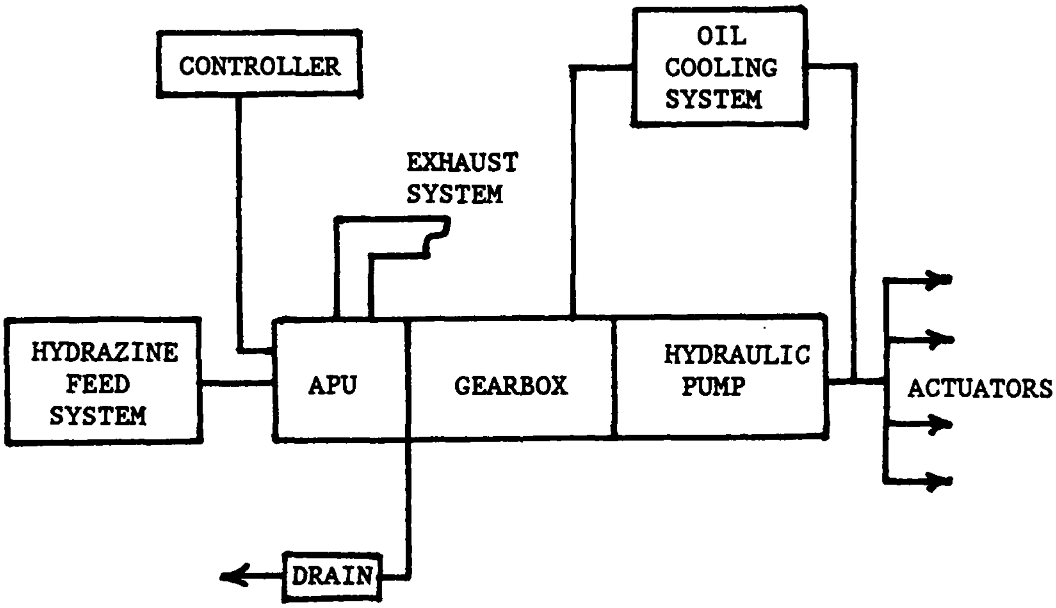


Figure 41. Current System

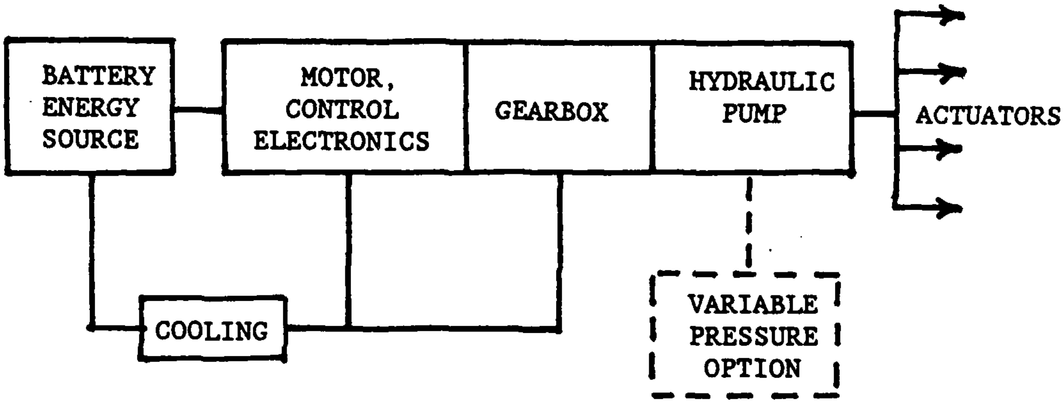


Figure 42. Battery System

density (HED) fuel cells (Figure 43). The fuel cell-based concept more closely resembles the current system since cooling requirements are normally accommodated via self-contained regulatory systems in the fuel cell, avoiding the need for a separate dedicated cooling system. However, the use of HED fuel cells avoids the need for toxic fuels such as hydrazine. Both the battery and HED fuel cell concepts include the optional use of variable pressure hydraulic pumps. While increasing costs, this option potentially reduces cooling requirements and provides the opportunity for substantial weight reductions.

Specific alternatives had been identified for each function or subsystem of each concept with the exception of the gearbox and the hydraulic pumps. While differences may exist in alternatives for these two components, their designs were deemed sufficiently similar to make any differences negligible for the purposes of this research relative to the significant differences in the other component designs. The exception to this generalization was the issue of whether or not to pursue the variable pressure pump option. Alternatives considered for the remaining subsystems were:

Battery energy sources:

- $Li\text{-}SO_2$
- $Li\text{-}SOCl_2$
- Ag-Zn (Primary)
- Ag-Zn (Secondary)
- Lead Acid
- $Ag\text{-}H_2$ (CPV)
- $Ag\text{-}H_2$ (IPV)

The first three batteries are referred to as primary batteries, meaning that they are used once and then replaced. The last four are secondary batteries, meaning that they can be re-charged up to a

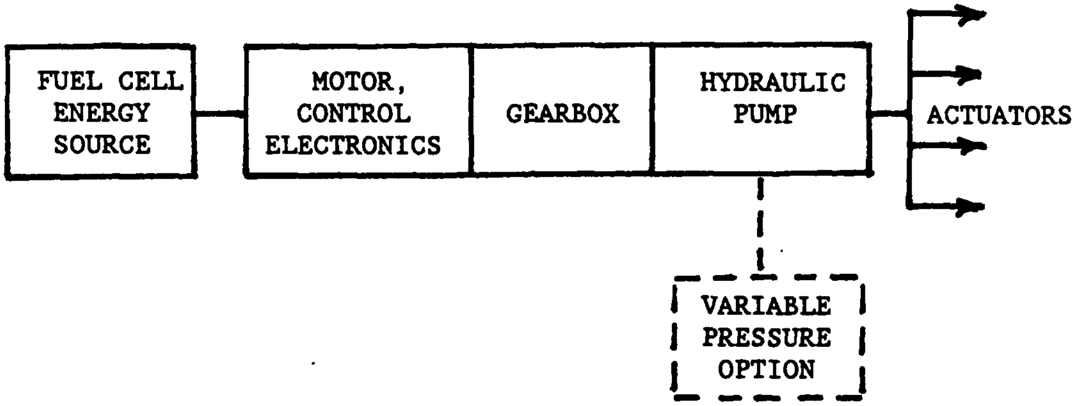


Figure 43. Fuel Cell System

certain life expectancy. The CPV and IPV designations for $Ag\text{-}H_2$ batteries stand for "common pressure vessel" and "individual pressure vessel", respectively, indicating whether or not each individual battery cell has its own container. While the former is somewhat cheaper and lighter, the latter improves safety and survivability by increasing containment of failures.

Fuel cell energy sources:
- Alkaline electrolyte
- Solid polymer electrolyte (SPE)
- Monolithic solid oxide
- Fuel cell/battery combination

The last fuel cell alternative involves primary use of fuel cells to handle the majority of power demands with the use of a battery supplement to handle the portions of mission profiles which call for very brief, but unusually high peaks in power requirements.

Cooling systems:
- A modified version of the existing freon system
- Bi-phase wax
- Ammonia boiler
- New independent system

Motors:
- High-speed induction
- Samarium cobalt
- Switched reluctance

At the time of this research, preliminary test data and estimates were available for many characteristics of the design alternatives just described. However, most of these technologies were at a sufficiently early stage of development or testing that the preliminary estimates were considered arithmetic means about which different implementations would vary. Given the high potential value and importance of obtaining government contracts to develop, produce, and support these technologies it was expected that numerous

corporations (hereafter referred to as contractors) would pursue their development if and when sufficient interest is expressed by NASA. NASA personnel perceive that the differences among the designs presented by the various contractors would be sufficiently important to merit their explicit consideration in the decision process.

After substantial consideration it was determined that a reasonable approach to achieve this source of variation was through the use of what we shall call contractor or design profiles. It was anticipated that, in general, the alternative designs for the APU energy source, cooling, and motor subsystems might be characterized as management oriented, technically oriented, or neutral. Management oriented designs are those which emerge from more established, conservative approaches to design. Such alternatives may be characterized by features such as lower cost and higher reliability and safety. By contrast, technically oriented designs are those which result from more innovative, state-of-the-art approaches. These designs typically trade off the previously mentioned characteristics for improved technological performance. The neutral profile is an attempt to compromise the two extremes. Presumably, the more specific the guidance NASA provides to designers, the more narrow the range of this source of variation. However, it is expected that sufficient leeway would always exist for the designing organization to be able to orient their design in a manner which reflects their own interpretations, preferences, and strategies.

Clearly, it is feasible that at more mature stages of development, other profiles may emerge. For example, ideally, one design may prove superior in all regards. However, should this occur, sophisticated analysis becomes unnecessary. The issue, then, is which design is best if a dominant alternative does not emerge. Therefore, the three design profiles described were explicitly considered for each power source, cooling, and motor alternative. Essentially, then, the mean characteristic estimates were represented as the neutral profile and were augmented with alternatives which displayed each of the other two specified design orientations.

Synthesis of candidate systems was then accomplished as suggested by Ostrofsky (1977d) by forming all possible combinations of subsystem alternatives for each concept, resulting in 4,753 candidate systems as follows:

Current system: 1 candidate system

Battery-powered systems: 7 battery options * 3 battery design profiles * 4 cooling system options * 3 cooling design profiles * 3 motor options * 3 motor design profiles * 2 variable pump options = 4,536 candidate systems

Fuel cell-powered systems: 4 fuel cell options * 3 fuel cell design profiles * 3 motor options * 3 motor design profiles * 2 variable pump options = 216 candidate systems

Total = 1 + 4,536 + 216 = 4,753 candidate systems

Screening of Candidate Systems

Since the synthesis of candidate systems is accomplished from a

relatively unconstrained point of view, this step is intended to enhance the ability of the analyst to evaluate the set of candidates by eliminating those which clearly cannot or should not be considered further. The three specific screens recommended by Ostrofsky (1977d) are those of physical realizability, economic worthwhileness, and financial feasibility.

With respect to physical realizability, NASA personnel acknowledged that future testing and evaluation might reveal some of the candidate systems to be unachievable. In particular, it was unclear whether the cooling requirements of some battery powered candidates could be accommodated solely by modifications to the existing freon system. However, no data was available which conclusively eliminated the possibility. Therefore, the principle of least commitment was invoked and none of the candidate systems were eliminated.

With respect to economic worthwhileness, opinions differed slightly among NASA personnel. Proponents of each concept or specific alternative tended to be somewhat more pessimistic about the worthwhileness of other candidates. However, all candidate systems appeared to have supporters. In addition, the determination of worthwhileness in state-of-the-art design efforts such as this is confounded by the possibility of spin-off applications which might enhance the worth of an otherwise inferior alternative. Since all

candidates identified were perceived worthwhile both for the intended purpose and for possible future applications, all were retained.

Finally, the determination of financial feasibility is typically based on budget or other resource constraints. In this case, no overall budget has been established since the current and previous studies have been exploratory in nature. At this stage, the evaluation effort is proceeding one step at a time with each funding decision based on both resource availability and the perceived importance or merit of the resulting design. Consequently, NASA personnel were unable to state unequivocally that any of the candidates identified would necessarily be financially infeasible. Again, the principle of least commitment mandated that the defined set of candidate systems remain intact based on the information currently available.

Preliminary Activities

Preparation for Analysis

This step in the design morphology provides at least two significant opportunities. The first is to establish a period of review and evaluation which is particularly helpful in linking the activities of the Feasibility Study to those of the Preliminary Activities when some period of time has elapsed between the two. In this demonstration, no such time lapse occurred. The second

significant opportunity is that of gaining insight into the set of candidate systems by examining possible groupings and evaluating each group's apparent advantages and disadvantages. The following paragraphs briefly review the most significant insights perceived by NASA personnel and the author.

The strongest perceived grouping of the candidate systems was that reflected in the two major concepts: delineation by energy source. Throughout the analysis it was apparent that NASA perceived the system weight to be a crucially important performance characteristic. The logic of this concern was quite apparent since the purpose of the shuttle is to transport payload, making weight an important constraint on the extent to which its mission can be accomplished. Both concepts were generally expected to produce weight reductions. However, fuel cell-powered candidates were expected to have a slight weight advantage, primarily due to the reduced cooling requirements. Fuel cell-powered alternatives also offer the advantages of R&D synergism with the Strategic Defense Initiative (SDI) and good safety characteristics, but are based on less mature technology and are expected to be somewhat more complex, with greater aircrew operations and groundcrew servicing requirements than battery-powered alternatives. Another tradeoff is that the replacement of failed fuel cells would be much more expensive than battery replacement.

Among battery types, secondary batteries, while costing more than

primary batteries, offer the advantages of increased life expectancy, a more mature stage of development, and generally improved safety characteristics.

Another significant tradeoff, mentioned earlier, is that candidates with the variable pressure pump option were expected to cost more but produce substantial weight reductions.

Advantages and disadvantages based on other groupings appeared less clear-cut and more difficult to generalize.

Definition of Criteria

Criteria

As the discussion thus far indicates, the choice of criteria was relatively straightforward. Beginning with the needs analysis and reinforced through the identification and formulation of the problem, four predominant decision criteria emerged:

x_1 - Cost
x_2 - Performance
x_3 - Availability
x_4 - Safety

By forming all possible combinations, these four marginal criteria lead to the synthesis of 11 possible criteria intersections:

x_{12} - Cost/Performance
x_{13} - Cost/Availability
x_{14} - Cost/Safety
x_{23} - Performance/Availability
x_{24} - Performance/Safety

x_{34} - Availability/Safety
x_{123} - Cost/Performance/Availability
x_{124} - Cost/Performance/Safety
x_{134} - Cost/Availability/Safety
x_{234} - Performance/Availability/Safety
x_{1234} - Cost/Performance/Availability/Safety

Relative Weights

Perhaps the most enlightening step accomplished in the demonstration was that of specifying relative weights for the criteria. Two characteristics of the weighting process were quite novel to the NASA personnel, eventually proving to be quite exciting and relevant to their decision process.

First was the possibility of relative weights varying throughout the possible range of criteria performance (X_i). Initially, it was not apparent to NASA personnel how this could be the case. However, during extended discussions it became apparent to both decision makers and the author that their basis for expressing preferences did, indeed, depend on the quality of candidate system performance. For example, cost (x_1) only became substantially important as the cost of the system increased. As long as the system's cost was relatively good (low), their preferences were primarily based on the other criteria. However, as cost increased (performance on x_1 decreased), their concern for this criterion increased exponentially. In addition, changes in relative importance were not always found to be monotonically increasing or decreasing throughout the X_i range. The importance of availability (x_3) for example was found to be

overshadowed by that of other criteria at both extremes of the range so that it reached its zenith in the middle of the X_i range.

The second important characteristic of the weighting process concerned the weighting of criteria intersections. Initially, the meaning of intersection relative weights proved quite elusive to the decision makers. Therefore, the author was led to innovate new approaches to describing and eliciting these weights. The vehicle which proved most useful in this regard was a simple 2 X 2 matrix with the name of one criterion in each cell. Presented with this matrix, NASA personnel were first asked to circle that cell or combination of cells which best described the most important basis (or decision rule) they would use in selecting from among candidate systems. Clearly, the natural first tendency might be to circle all four cells. Therefore, it was important to point out that a preference for good performance on multiple criteria might (and probably would) incur sacrifices on performance of subsets of that combination. In other words, by preferring a combination of cost and performance they would be indicating a willingness to sacrifice some measure of either cost or performance in order to achieve a better combination or balance of the two.

The simultaneous consideration of both of these characteristics (variance throughout the range and consideration of intersections) required repeated iterations through the weighting process. Ultimately, a high degree of consistency was obtained at which point

both the decision makers and the author felt that the derived relative weights (Figures 44-50) represented the true preference structure with a high degree of accuracy. Various test comparisons of hypothetical candidate systems supported the validity of these relative weights.

Note that of the eleven possible criteria intersections, three first-order intersections (x_{13}, x_{23}, and x_{24}) were identified as having significant importance: the others were not considered important. However, it is also interesting to note that two of the intersections (Performance/Availability and Performance/Safety) were deemed to be generally more important than the marginal criteria through much of the X_i range.

The relative weights shown in Figures 44-50 reflect the following functional relationships:

$$a_1 = \begin{cases} 1 \text{ if } X_i \leq .01 \\ |(\log Xi)/2| \text{ if } X_i > .01 \end{cases} \qquad \text{Eq. 5.1}$$

$$a_2 = \begin{cases} .5 \text{ if } X_i \leq .5 \\ X_i \text{ if } X_i > .5 \end{cases} \qquad \text{Eq. 5.2}$$

$$a_3 = 2X_i - 2X_i^2 \qquad \text{Eq. 5.3}$$

$$a_4 = \begin{cases} 1 - X_i \text{ if } X_i \leq .5 \\ .5 \text{ if } X_i > .5 \end{cases} \qquad \text{Eq. 5.4}$$

$$a_{13} = \begin{cases} .3 \text{ if } X_i \leq .5 \\ .6 - (3X_i/5) \text{ if } X_i > .5 \end{cases} \qquad \text{Eq. 5.5}$$

$$a_{23} = .7 \qquad \text{Eq. 5.6}$$

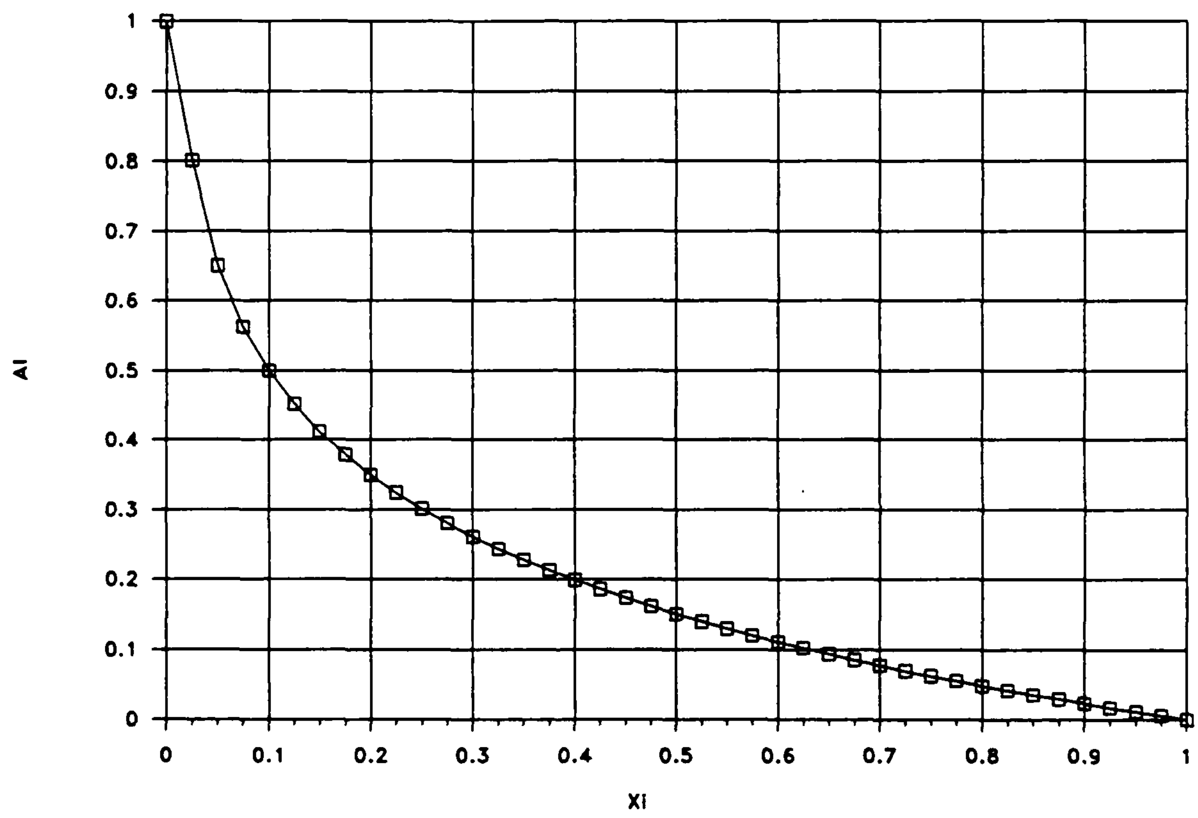


Figure 44. Cost Relative Weight

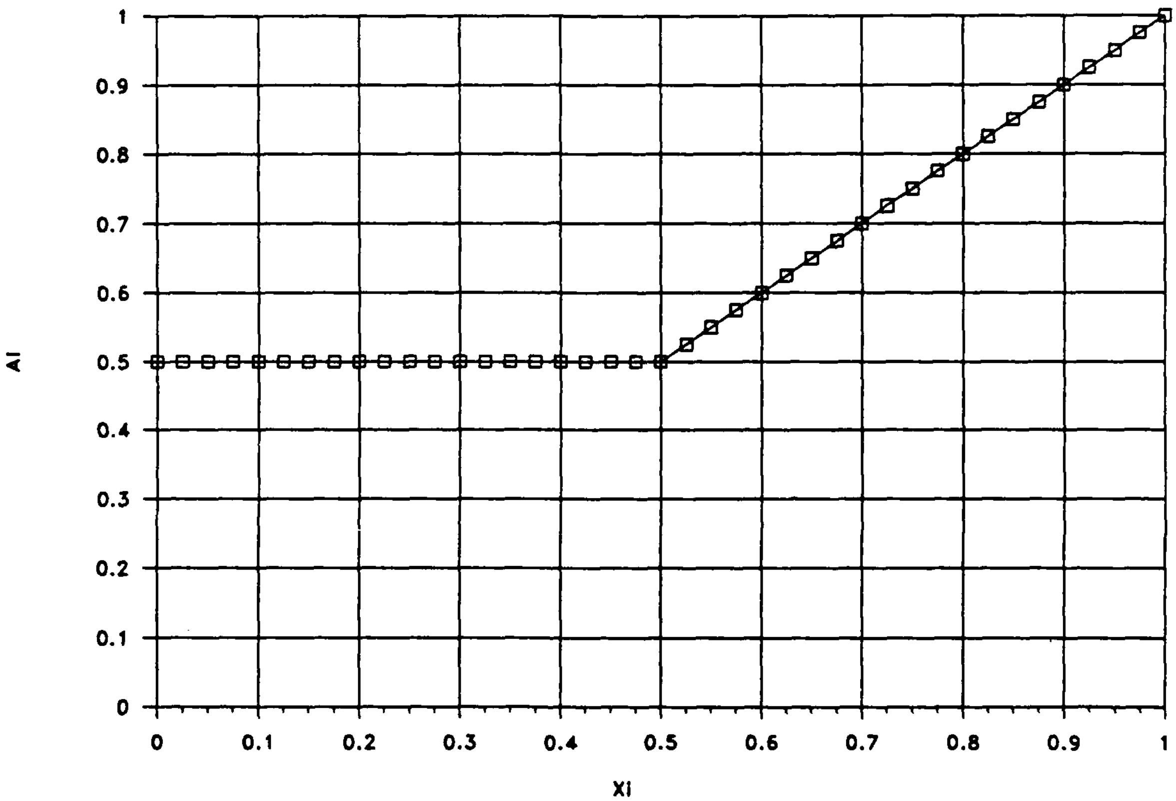


Figure 45. Performance Relative Weight

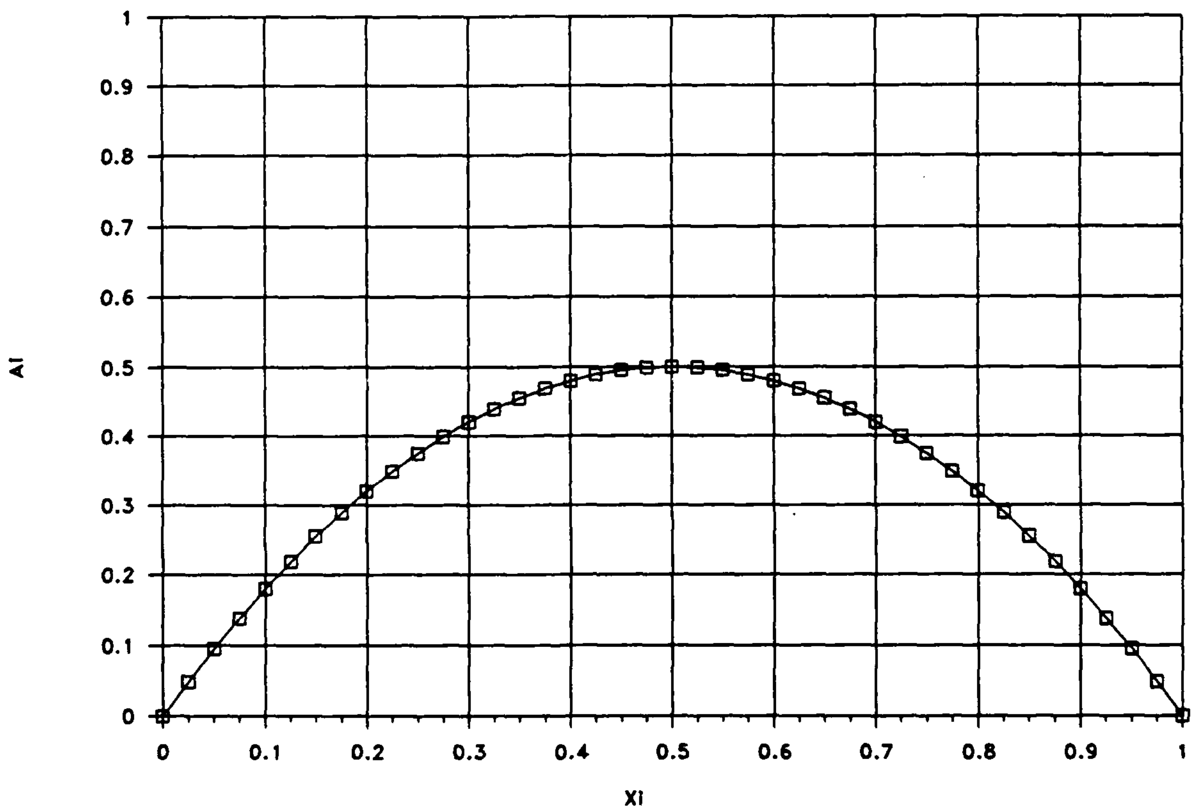


Figure 46. Availability Relative Weight

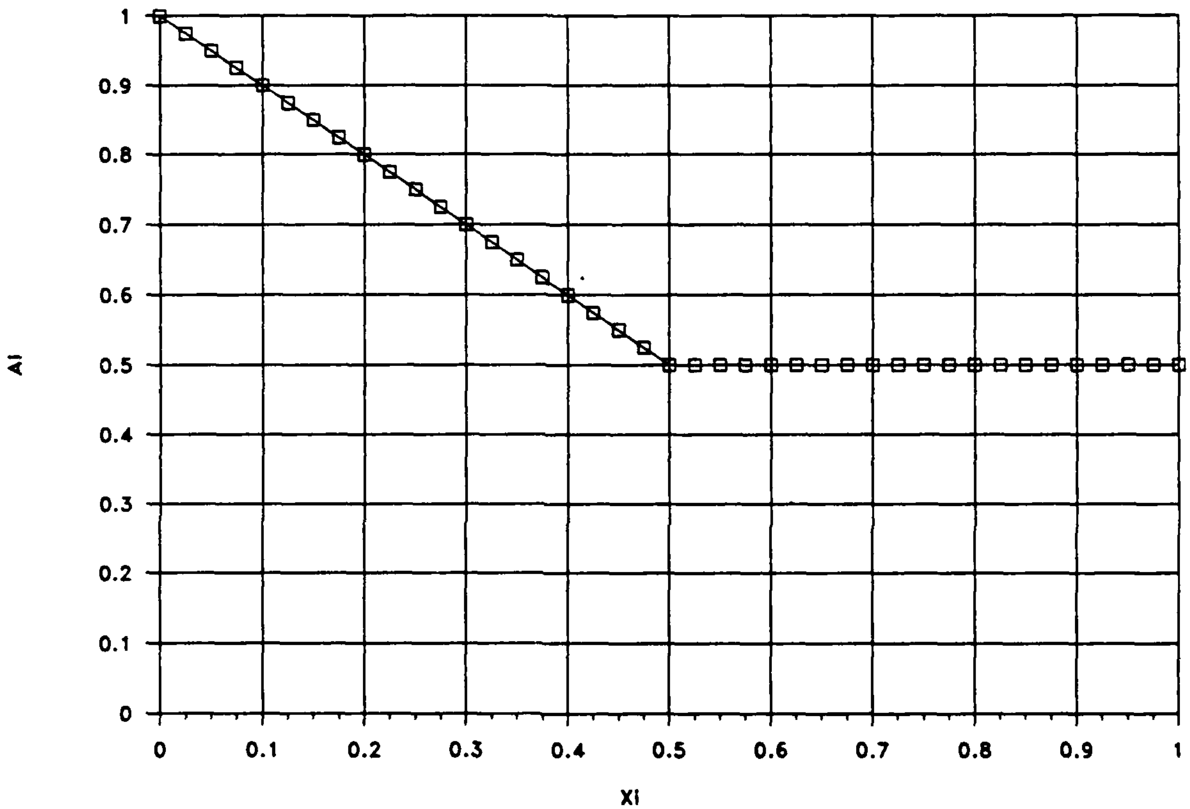


Figure 47. Safety Relative Weight

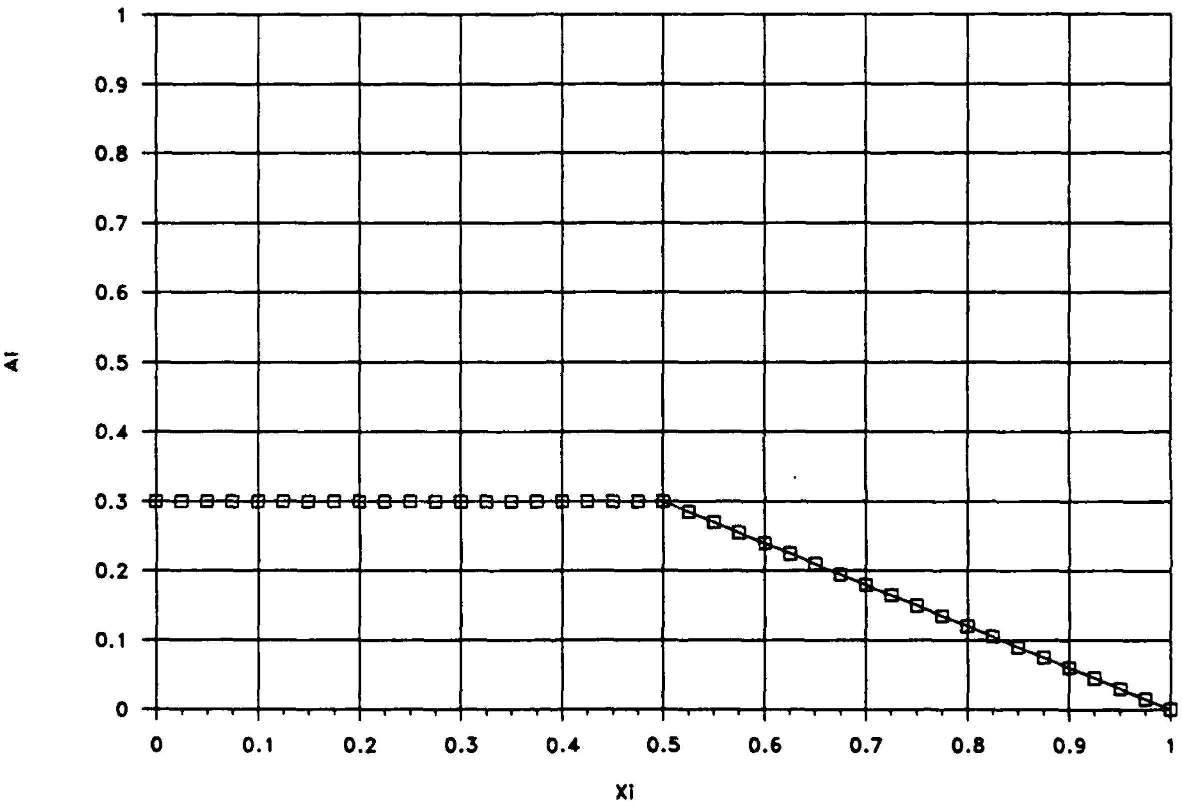


Figure 48. Cost/Availability Relative Weight

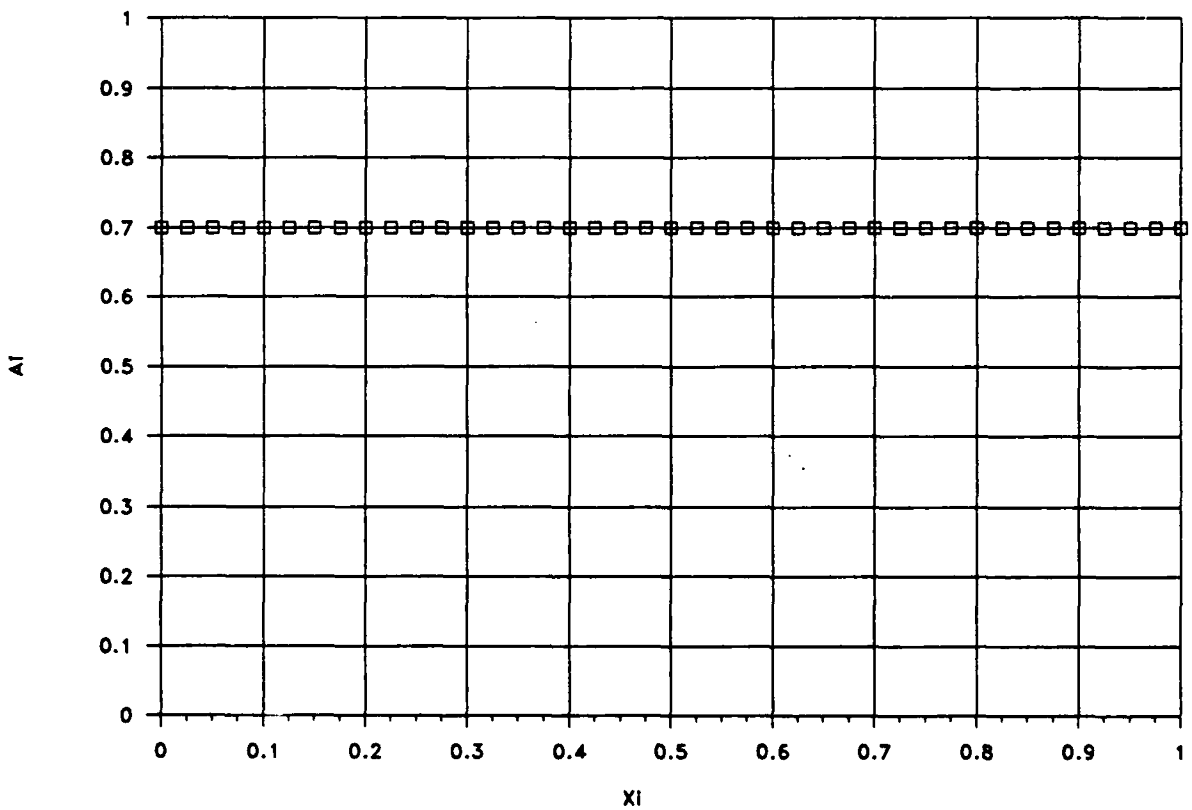


Figure 49. Performance/Availability Relative Weight

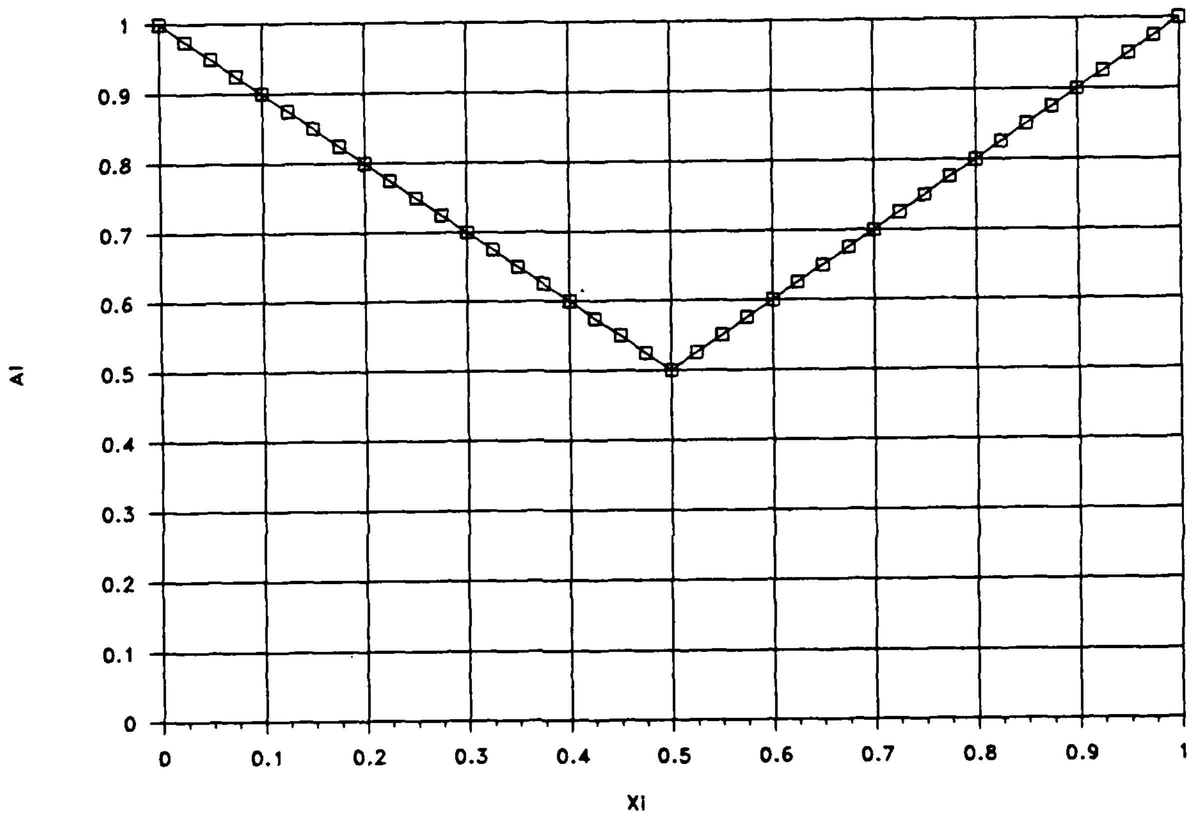


Figure 50. Performance/Safety Relative Weight

$$a_{24} = \begin{cases} 1-X_i \text{ if } X_i \leq .5 \\ X_i \text{ if } X_i > .5 \end{cases} \qquad \text{Eq. 5.7}$$

All other $a_i=0$.

Although many of these functions contain inflection points, none involve discontinuities. Therefore, the use of Model VII is indicated.

Definition of Parameters

In order to obtain figures of merit for the criteria identified (and their intersections) it was necessary to specify the constituent elements of each to a level of detail which would facilitate some form of measurement. Numerous iterations throughout the course of the analysis produced the criterion elements shown in Tables 16-19. Each element will be explained in the section on Criterion Modeling.

As described in Chapter 2, elements were coded as "a" if they could be directly measured via some (preferably objective) scale. These elements are hereafter referred to as parameters and are denoted as y_k. A total of 41 parameters emerged for this demonstration. Thirty-four of these parameters were deemed measurable using objective units. Standard subjective scales were established for the remaining seven parameters.

Twenty-six elements in Tables 16-19 were coded "b". These will

TABLE 16.

COST ELEMENTS

Criterion x_1: Cost

Element	Code
Expected initial cost	b
Initial cost estimate	b
Gross acquisition cost	a
Risk factor	b
Maturity of technology	a
Contractor risk	b
Battery contractor success	a
Battery contractor attempts	a
Fuel cell contractor successes	a
Fuel cell contractor attempts	a
Motor contractor successes	a
Motor contractor attempts	a
Cooling system contractor successes	a
Cooling system contractor attempts	a
Recurring costs	b
Replacement costs	b
Number of replacement parts required	b
Number of missions planned	a
APU operating hours per mission	a
System life expectancy	a
Unit cost to replace	b
Cost of new/refurbished unit	a
Serial hours for replacement	a
Parallel hours for replacement	a
Cost of serial hours	a
Cost of parallel hours	a
Servicing costs	b
Number of servicing actions required	b
Time between servicing	a
Cost of servicing action	b
Quantity of consumables for servicing	a
Cost of consumables	a
Serial hours for servicing	a
Parallel hours for servicing	a
Long term costs	b
Disposal/retirement costs	a
Savings due to future applications/ extensions	a

TABLE 17.

PERFORMANCE ELEMENTS

Criterion x_2: Performance

Element	Code
Physical requirements	b
Weight	a
Volume	a
Operating temperature range	a
Impact on operations	b
Crew operations requirements	a
Complexity	a
Integration effects	b
Impact on performance of other systems	a
Ability to facilitate beneficial changes	a

TABLE 18.

AVAILABILITY ELEMENTS

Criterion x_3: Availability

Element	Code
Initial availability	b
Acquisition time	a
Installation time	a
Risk factor	b
Operational availability	b
Total possessed calendar hours	b
Mission length	a
Time between missions	a
Down time hours	b
Maintenance down time	b
System life expectancy	a
Time to replace	b
Serial hours for replacement	a
Parallel hours for replacement	a
Time between servicing	a
Time to service	b
Serial hours for servicing	a
Parallel hours for servicing	a
Mean operating time between maintenance	b
Probability of mission completion	b
Number of APU systems installed	a
Number of APU's for mission completion	a

TABLE 19.

SAFETY ELEMENTS

Criterion x_4: Safety

Element	Code
Flight safety	b
Tolerance to abuse	a
Potential severity of failure	a
Probability of mission completion	b
Ground safety	b
Hazardous handling requirements	a
Maintenance down time	b
Mean operating time between maintenance	b

be referred to as submodels (or simply models) and denoted as z_j. The next section describes each submodel, its constituent parameters, and the parameter units of measure.

Subscripts (1-41) were assigned sequentially to the parameters in the order shown in Table 20. However, since more than one "layer" of submodels was frequently necessary to relate parameters to criteria, subscripts for submodels were assigned to reflect the hierarchical relationship between the submodel and its parent criterion. For example, z_{32} is the second first-level submodel used to compute criterion x_3. Similarly, z_{321} is the first submodel used in the computation of z_{32}.

As a means of helping to insure consistency, completeness, and compactness of the parameter set, Ostrofsky (1977d) has suggested the construction of a table relating parameters to criteria through submodels. Unfortunately, the large number of parameters and submodels used in this demonstration makes the depiction of such a table problematic. Therefore, two such tables have been constructed. The first, Table 20, relates parameters directly to the highest level submodel for each criterion. This table provides a useful vehicle for identifying missing, unnecessary, or ambiguous parameters. The second table, Table 21, relates all submodels to the criteria in which they are used.

At this point, it is appropriate to point out that a complete set

TABLE 20.

RELATION OF PARAMETERS TO SUBMODELS

	x_1			x_2			x_3			x_4	
Parameters	z_{11}	z_{12}	z_{13}	z_{21}	z_{22}	z_{23}	z_{31}	z_{32}	z_{33}	z_{41}	z_{42}
1 Gross acq cost	X										
2 Maturity of tech	X						X				
3 Battery successes	X						X				
4 Battery attempts	X						X				
5 Fuel cell successes	X						X				
6 Fuel cell attempts	X						X				
7 Motor successes	X						X				
8 Motor attempts	X						X				
9 Cooling successes	X						X				
10 Cooling attempts	X						X				
11 # missions planned		X									
12 APU OH per mission		X									
13 System life expect		X						X	X	X	X
14 Cost of new unit		X									
15 Serial hrs replace		X						X			X
16 Parallel hrs replace		X						X			X
17 Cost of serial hrs		X									
18 Cost of parallel hrs		X									
19 Time between service		X						X			X
20 Qty of consumables		X									
21 Cost of consumables		X									
22 Serial hrs service		X						X			X
23 Parallel hrs service		X						X			X
24 Dispose/retire costs			X								
25 Future appl/ext			X								
26 Weight				X							
27 Volume				X							
28 Ops temp range				X							
29 Crew ops reqmts					X						
30 Complexity					X						
31 Impact other systems						X					
32 Facilitate good chgs						X					
33 Acquisition time							X				
34 Installation time							X				
35 Mission length								X	X	X	
36 Time betw missions								X			
37 # APU's installed									X	X	
38 # APU's required									X	X	
39 Tolerance to abuse										X	X
40 Severity of failure										X	X
41 Hazardous handling											X

TABLE 21.

RELATION OF SUBMODELS TO CRITERIA

	x_1			x_2			x_3			x_4	
<u>Submodels</u>	z_{11}	z_{12}	z_{13}	z_{21}	z_{22}	z_{23}	z_{31}	z_{32}	z_{33}	z_{41}	z_{42}
Expected initial cost	X										
Initial cost estimate	X										
Risk factor	X						X				
Contractor risk	X						X				
Recurring costs		X									
Replacement costs		X									
# replacements required		X									
Unit cost to replace		X									
Servicing costs		X									
# svcing actions reqd		X									
Cost of svcing action		X									
Long term costs			X								
Physical requirements				X							
Impact on operations					X						
Integration effects						X					
Initial availability							X				
Ops availability								X			
Total possessed hrs								X			
Down time hours								X			
Maint down time								X			X
Time to replace								X			
Time to service								X			
Mean time between maint								X			X
Prob mission completion									X	X	
Flight safety										X	
Ground safety											X

of parameter values did not exist for the set of 4,753 candidate systems. While such data was clearly desirable, its a priori existence would be an unreasonable expectation since, as stated previously, the complete set of candidate systems was not synthesized prior to this research. Furthermore, it should be remembered that many of the subsystems considered currently exist in theory or early prototype form only. Therefore, it was necessary to combine the known data and estimates in as realistic a manner as possible pending the ability to verify such estimates through subsequent testing. This is a common characteristic of high technology projects. In fact, Dr. Aaron Cohen, Director of Johnson Space Center, has observed (1989) that the first principle of maximizing success in project management is that "you must fearlessly base your decisions on the best information available." To facilitate these estimates, the method referred to by Tversky and Kahneman (1974) as the "anchoring and adjustment heuristic" was employed. This method is accomplished by first identifying a known reference point ("anchor") and then adjusting for the effects (both marginal and interactive) of the other relevant factors. By having used this approach, we are not only able to proceed with the analysis based on the best existing information, but have also identified those data requirements which should serve as the basis for subsequent test and evaluation activities.

Criterion Modeling

The following paragraphs describe the way in which measurable

parameter values were combined for this demonstration to produce meaningful figures of merit for each criterion. In some cases, validated theoretical or logical models are available or apparent to serve as a guide. In other cases, the analyst must determine a meaningful method of combination. In such cases, the key is to produce a metric which responds to candidate system differences in a manner which unambiguously reflects better or worse performance, even though the resulting units of measure themselves may not display any apparent meaning. To insure consistency, the models for this demonstration were constructed in such a way that, even if lower submodel values were good, models at the criterion level would insure that higher raw criteria scores always reflected better performance than low criteria scores.

Before introducing the submodels, it is appropriate to cite the following caveats. First, it is recognized that, as with any modeling effort, the models could be made more detailed. Particularly when dealing with complex systems, there is virtually no end to the depth to which models can be formulated. In addition, it is acknowledged that the models used might be made more rigorous. For example, many of the metrics formed here by taking simple products might have theoretical alternatives. However, both of these considerations must be traded off against both the accuracy of the data and the objectives of the research. In this regard, the models presented and used were deemed appropriate to adequately assess the relative performance of the candidate systems and, more importantly for the purposes of this

research, to facilitate the demonstration of the proposed method for criterion function computation.

x_1 - Cost

This model reflects cost performance by taking the inverse of the sum of net primary life cycle costs. The constituent elements are:

z_{11} - Expected initial cost

z_{12} - Recurring costs

z_{13} - Long term costs

All values were converted to millions of dollars prior to inclusion in this model. The expression used to compute x_1 is:

$$x_1=1/(z_{11}+z_{12}+z_{13}) \qquad \text{Eq. 5.8}$$

z_{11} - Expected Initial Cost

This model reflects the anticipated initial costs of development and implementation. Experienced NASA personnel recognize that not all initial estimates prove to be equally valid. Therefore, the model explicitly includes a risk factor which inflates the initial estimate in a manner which reflects the contractor's (developer/producer) "track record" in previous design efforts. The constituent elements are:

z_{111} - Initial cost estimate (Eq. 5.10)

z_{112} - Risk factor (Eq. 5.11)

The resulting expression is:

$$z_{11}=z_{111}*z_{112} \qquad \text{Eq. 5.9}$$

z_{111} - Initial cost estimate

Normally, this submodel would include the various inputs to initial implementation costs, e.g. R&D, production, raw materials, direct labor, burden, etc. However, in prior EAPU studies, total gross initial cost estimates had been provided to NASA by Rockwell without the underlying breakout. Therefore, while it was deemed prudent to explicitly identify the existence of a submodel, the expression used for this demonstration was simply:

$$z_{111}=y_1 \quad \text{Eq. 5.10}$$

where y_1 is the gross acquisition cost measured in millions of dollars.

z_{112} - Risk factor

As mentioned previously, this submodel attempts to reflect the inflationary tendency typically experienced with risky alternatives. The constituent elements are:

y_2 - Maturity of technology
(0 - new/untested,
1 - laboratory testing,
2 - off-the-shelf)

z_{1121} - Contractor risk (Eq. 5.12)

The expression for z_{112} is:

$$z_{112}=1/(((y_2+2)/4)*z_{1121}) \quad \text{Eq. 5.11}$$

z_{1121} - Contractor risk

This submodel reflects the degree to which the prospective subsystem contractor (developer/producer) has succeeded in meeting comparable previous obligations. The constituent parameters are:

y_3 - Battery contractor successes (The number of comparable contract deliverables or milestones the contractor has achieved on time and successfully meeting specifications)

y_4 - Battery contractor attempts (The number of comparable contract deliverables or milestones the contractor has undertaken)

y_5 - Fuel cell contractor successes

y_6 - Fuel cell contractor attempts

y_7 - Motor contractor successes

y_8 - Motor contractor attempts

y_9 - Cooling system contractor successes

y_{10} - Cooling system contractor attempts

The submodel is computed as:

$$z_{1121}=((y_3/y_4)^{1/y_4})*((y_5/y_6)^{1/y_6})*((y_7/y_8)^{1/y_8})*((y_9/y_{10})^{1/y_{10}}) \quad \text{Eq. 5.12}$$

z_{12} - Recurring costs

This model identifies the expected operational costs due to replacement and servicing. The constituent submodels are:

z_{121} - Replacement costs (Eq. 5.14)

z_{122} - Servicing costs (Eq. 5.17)

The expression is:

$$z_{12}=z_{121}+z_{122} \qquad \text{Eq. 5.13}$$

$\underline{z_{121}}$ - Replacement costs

This model computes the expected costs of replacement due to failure. It includes:

z_{1211} - Number of replacements required (Eq. 5.15)

z_{1212} - Unit cost to replace (Eq. 5.16)

It is computed as:

$$z_{121}=z_{1211}*z_{1212} \qquad \text{Eq. 5.14}$$

$\underline{z_{1211}}$ - Number of replacements required

This submodel estimates the expected number of times the system will need to be replaced over the life of the shuttle program. It considers:

y_{11} - Number of missions planned (This parameter is a constant. Original NASA plans were targeted for 100 missions for each of 4 vehicles. However, the unexpectedly slow growth in the shuttle sortie rate would seem to make that figure optimistic, even with the possibility of delayed shuttle retirement. Therefore, an estimate of 300 was used.)

y_{12} - APU operating hours per mission (Although each mission may range from 7 to 30 days, the flight profile calls for APU operation only during ascent, descent, and landing. A constant of 1.5 operating hours was used.)

y_{13} - System life expectancy (operating hours)

The submodel is:

$$z_{1211}=(y_{11}*y_{12})/y_{13} \qquad \text{Eq. 5.15}$$

z_{1212} - Unit cost to replace

This submodel includes both the hardware replacement and repair time costs. The constituent parameters are:

y_{14} - Cost of new/refurbished unit (thousands of dollars)

y_{15} - Serial hours for replacement (Repair hours during which no other work can be simultaneously accomplished on the orbiter.)

y_{16} - Parallel hours for replacement (Repair hours during which other orbiter repairs or servicing can also be be accomplished in parallel.)

y_{17} - Cost of serial hours (Estimated at $28,700 per hour)

y_{18} - Cost of parallel hours ($19,000 per hour)

The submodel is:

$$z_{1212}=y_{14}+(y_{15}*y_{17})+(y_{16}*y_{18}) \qquad \text{Eq. 5.16}$$

z_{122} - Servicing costs

This model computes the expected costs due to required routine servicing. The logic of the model and its submodels parallels that of z_{121}. Constituent submodels for servicing costs are:

z_{1221} - Number of servicing actions required (Eq. 5.18)

z_{1222} - Cost of servicing action (Eq. 5.19)

The submodel is:

$$z_{122}=z_{1221}*z_{1222} \qquad \text{Eq. 5.17}$$

z_{1221} - Number of servicing actions required

This submodel computes the expected number of times the APU will require servicing. It is based on:

y_{11} - Number of missions planned (300)

y_{12} - APU operating hours per mission (1.5)

y_{19} - Time between servicing (All of the candidates identified required some form of servicing after every flight. Therefore, this parameter was treated as a constant of 1.5 operating hours.)

The submodel is:

$$z_{1221}=(y_{11}*y_{12})/y_{19} \qquad \text{Eq. 5.18}$$

(Note: It is recognized that, for the set of candidate systems identified in this demonstration, z_{1221} will always be 300.)

z_{1222} - Cost of servicing action

The constituent parameters are:

y_{17} - Cost of serial hours ($28,700)

y_{18} - Cost of parallel hours ($19,000)

y_{20} - Quantity of consumables for servicing (lbs.)

y_{21} - Cost of consumables ($/lb.)

y_{22} - Serial hours for servicing (hours)

y_{23} - Parallel hours for servicing (hours)

The model is:

$$z_{1222}=(y_{20}*y_{21})+(y_{22}*y_{17})+(y_{23}*y_{18}) \qquad \text{Eq. 5.19}$$

z_{13} - Long term costs

This submodel computes the expected net cost of longer term considerations. It includes:

y_{24} - Disposal/retirement costs (millions of dollars)

y_{25} - Savings due to future applications/extensions (Specific opportunities identified included EMA, use on solid rocket boosters, and SDI. Measured in millions of dollars.)

The expression is:

$$z_{13}=y_{24}-y_{25} \qquad \text{Eq. 5.20}$$

x_2 - Performance

This criterion considers a variety of relevant technical performance measures. It should be noted that other relevant performance measures which might normally be included in the evaluation of a new APU design may not be included here since they are constrained by the requirements of the existing space shuttle and would therefore be included as required specifications. Since the measures used here reflect less desirable performance as the units increase, the criterion measure takes the inverse of their product. The pertinent elements are:

z_{21} - Physical requirements (Eq. 5.22)

z_{22} - Impact on operations (Eq. 5.23)

z_{23} - Integration effects (Eq. 5.24)

The expression is:

$$x_2=1/(z_{21}*z_{22}*z_{23}) \qquad \text{Eq. 5.21}$$

z_{21} - Physical requirements

This submodel conglomerates the more important physical features of the system. Its elements are:

y_{26} - Weight (lbs.)

y_{27} - Volume (cubic feet)

y_{28} - Operating temperature range (degrees C)

The submodel is:

$$z_{21}=((y_{26})^3*y_{27})/y_{28} \qquad \text{Eq. 5.22}$$

Note that y_{26} has been raised to the third power to reflect its perceived significance relative to the other two parameters.

z_{22} - Impact on operations

It is desirable that a new system be as transparent and/or simple to operate and support as possible both on the ground and in flight. This submodel attempts to assess the magnitude of potential impacts in terms of:

y_{29} - Crew operations requirements (Number of minutes of direct crew attention or manipulation required in flight.)

y_{30} - Complexity
(1 - Very simple for both ground and flight operations
2 - Somewhat complex either in flight or on ground
3 - Somewhat complex both in flight and on ground
4 - Very complex either in flight or on ground
5 - Very complex both in flight and on ground)

The model is:

$$z_{22}=y_{29}*y_{30} \qquad \text{Eq. 5.23}$$

z_{23} - Integration effects

This model explicitly recognizes that integration of a new APU system into the existing orbiter may have good and/or bad effects on other orbiter systems. The model therefore considers:

y_{31} - Adverse impact on performance of other systems
(1 - Little or no impact on other systems
2 - Moderate impact on other systems
3 - Significant impact on other systems)

y_{32} - Ability to facilitate beneficial changes
(1 - Does nothing to facilitate beneficial changes
2 - Potentially minor beneficial changes facilitated
3 - Potentially major beneficial changes facilitated)

The submodel is:

$$z_{23}=y_{31}/y_{32} \qquad \text{Eq. 5.24}$$

x_3 - Availability

This criterion simultaneously considers three different measures of the extent to which the system will be operational when needed. The constituent submodels are:

z_{31} - Initial availability (Eq. 5.26)

z_{32} - Operation availability (Eq. 5.27)

z_{33} - Probability of mission completion (Eq. 5.35)

The expression is:

$$x_3=(z_{32}*z_{33})/z_{31} \qquad \text{Eq. 5.25}$$

z_{31} - Initial availability

This submodel assesses how soon the system could be made available for operational use. Since, like initial costs, this estimate typically proves to be understated, it is modified by the risk factor identified previously. The model considers:

y_{33} - Acquisition time (years)

y_{34} - Installation time (weeks)

z_{112} - Risk factor (Eq. 5.11)

The submodel is:

$$z_{31}=(y_{33}+(y_{34}/52))*z_{112} \qquad \text{Eq. 5.26}$$

(Note: To avoid division by zero in Eq. 5.25, the results of this submodel were arbitrarily inflated by one year. This modification simply facilitated computation and had no effect on the relative performance of the candidate systems since it added the same constant to all candidates.)

z_{32} - Operational availability

Essentially, operational availability measures the portion of time a system will be available for use. This model and its associated submodels follow Seger's (1989) revision of the traditional computation (Blanchard, 1986) in order to accommodate the difference between systems such as the APU, which are operated intermittently, and those which are operated continuously. It utilizes:

z_{321} - Total possessed calendar hours (Eq. 5.28)

z_{322} - Down time hours (Eq. 5.29)

The model is:

$$z_{32}=(z_{321}-z_{322})/z_{321} \qquad \text{Eq. 5.27}$$

z_{321} - Total possessed calendar hours

This model simply determines the duration of an operating cycle (the time from the start of one use until the start of the next use). It includes:

y_{35} - Mission length (operating hours)

y_{36} - Time between missions (A mean estimate of 6 months was selected as a compromise between current and intended usage.)

The model is the sum of the two:

$$z_{321}=y_{35}+(y_{36}*30*24) \qquad \text{Eq. 5.28}$$

z_{322} - Down time hours

This model computes the expected length of down time incurred by servicing and/or replacement per operating cycle. It is based on:

y_{35} - Mission length (operating hours)

z_{3221} - Maintenance down time (Eq. 5.30)

z_{3222} - Mean operating time between maintenance (Eq. 5.33)

The submodel is:

$$z_{322}=(y_{35}*z_{3221})/z_{3222} \qquad \text{Eq. 5.29}$$

z_{3221} - Maintenance down time

This submodel follows Blanchard's (1986) method for determining the average portion of the time the system will be unavailable due to replacement and/or servicing actions. The constituent elements are:

y_{13} - System life expectancy (operating hours)

z_{32211} - Time to replace (Eq. 5.31)

y_{19} - Time between servicing (operating hours)

z_{32212} - Time to service (Eq. 5.32)

The model is computed as:

$$z_{3221}=((z_{32211}/y_{13})+(z_{32212}/y_{19}))/((1/y_{13})+(1/y_{19})) \quad \text{Eq. 5.30}$$

z_{32211} - Time to replace

This submodel reflects the total time for a replacement action. It consists of:

y_{15} - Serial hours for replacement (hours)

y_{16} - Parallel hours for replacement (hours)

The model is simply:

$$z_{32211}=y_{15}+y_{16} \quad \text{Eq. 5.31}$$

z_{32212} - Time to service

Similarly, this submodel totals the time required for a servicing action. The relevant times are:

y_{22} - Serial hours for servicing

y_{23} - Parallel hours for servicing

The submodel is:

$$z_{32212} = y_{22} + y_{23} \qquad \text{Eq. 5.32}$$

z_{3222} - Mean operating time between maintenance

This submodel follows Blanchard (1986) based on:

y_{13} - System life expectancy (operating hours)

y_{19} - Time between servicing (operating hours)

It is computed as:

$$z_{3222} = 1/((1/y_{13})+(1/y_{19})) \qquad \text{Eq. 5.33}$$

z_{33} - Probability of mission completion

Whereas operational availability simply assesses an overall mean percentage of (essentially calendar) availability, this submodel attempts to assess the potential that the APU's may fail to operate to the extent required on each specific mission. It follows the frequently used model presented by Blanchard (1986). Experience indicates that the time between failures for electro-mechanical devices typically approximates a Weibull distribution. The form of the Weibull distribution is described as follows (Besterfield, 1979):

$$f(t) = \alpha\beta(t-\gamma)^{\beta-1} e^{-\alpha(t-\gamma)^{\beta}} \qquad \text{Eq. 5.34}$$

With the exception of expected means, the preliminary nature of the estimates used in this demonstration precluded the availability of

empirical distributions which would be useful in providing meaningful estimates of the remaining Weibull parameters. Therefore, to facilitate comparison of candidate systems, it was arbitrarily assumed that $\beta=1$ and $\gamma=0$, in which case the Weibull reduces to the exponential distribution. (Consequently, the failure rate is assumed Poisson.) Submodel z_{33} uses:

y_{13} - System life expectancy (operating hours)

y_{35} - Mission length (operating hours)

y_{37} - Number of APU systems installed (3)

y_{38} - Number of APU systems required for mission completion (2)

The model is computed as:

$$z_{33} = \sum_{x=0}^{y_{37}-y_{38}} \frac{((y_{37}*y_{35})/y_{13})^{x} * e^{-((y_{37}*y_{35})/y_{13})}}{x!} \qquad \text{Eq. 5.35}$$

x_4 - Safety

This criterion considers the perceived level of safety resulting from the design in terms of both flight and ground operations. The constituent submodels are:

z_{41} - Flight safety (Eq. 5.37)

z_{42} - Ground safety (Eq. 5.38)

Safety is computed as:

$$x_4 = z_{41} * z_{42} \qquad \text{Eq. 5.36}$$

z_{41} - Flight Safety

This sumbmodel treats flight safety as a function of the likelihood of failure as well as its potential consequences. It includes:

y_{39} - Tolerance to abuse
(1 - Fragile, easily damaged
2 - Reasonably tolerant if treated as designed
3 - Highly tolerant, even if abused)

y_{40} - Potential severity of failure
(1 - Consequences of failure are not severe
2 - Failure may incapacitate components but is not expected to threaten life or mission
3 - Failure may incapacitate entire systems, threatening life and/or mission completion)

z_{33} - Probability of mission completion (Eq. 5.35)

The model is:

$$z_{41} = z_{33} * y_{39} / y_{40} \qquad \text{Eq. 5.37}$$

z_{42} - Ground safety

This model computes ground safety as a function of the risks faced by maintenance technicians as well as the frequency and duration of exposure to those risks. The model considers:

y_{39} - Tolerance to abuse (described for z_{41})

y_{40} - Potential severity of failure (described for z_{41})

y_{41} - Hazardous handling requirements
(1 - Little or none
2 - Low hazard infrequently required
3 - Low hazard frequently required
4 - High hazard infrequently required
5 - High hazard frequently required)

z_{3221} - Maintenance down time (Eq. 5.30)

z_{3222} - Mean operating time between maintenance (Eq. 5.33)

The submodel is:

$$z_{42}=(z_{3222}*y_{39})/(z_{3221}*y_{40}*y_{41}) \qquad \text{Eq. 5.38}$$

Formulation of the Criterion Function

Since the criteria relative weights are represented as continuous functions and since the need exists to consider intersection terms, Model VII (as described in Chapters 2 and 4) provides the appropriate form of the criterion function:

$$CF = \sum_{i=1}^{n} \delta_i\theta_i - \sum_{\substack{i \\ i\neq j}}^{n} \sum_{j}^{n} \delta_{ij}\theta_{ij}$$

$$+ \sum_{\substack{i \\ i\neq j \\ j\neq k \\ i\neq k}}^{n} \sum_{j}^{n} \sum_{k}^{n} \delta_{ijk}\theta_{ijk} \qquad \text{Eq. 5.39}$$

$$- \ldots \pm \sum_{\substack{i \\ i\neq j \\ \vdots \\ J\neq J+1}}^{n} \sum_{j}^{n} \ldots \sum_{J+1}^{n} \delta_{ijk\ldots(J+1)}\theta_{ijk\ldots(J+1)}$$

where

$$\delta_i\theta_i = \delta_i A_i F(x_i) \qquad \text{Eq. 5.40}$$

The $F(x_i)=X_i$, representing criteria performance, and are computed via the models in Eq. 5.8-5.38 and Eq. 4.46. The A_i (relative weights)

are obtained as described in Chapter 4, beginning with the functions $a_i = f_i(X_i)$ as defined in Eq. 5.1-5.7 or $a_i = 0$ otherwise.

Chapter 4 included a discussion of the use of empirical distribution functions as opposed to continuous approximations. Based on the reasoning presented in that discussion, the empirically observed distributions were used for the computations in this demonstration. Additional factors supporting their use were the sufficiently large number of candidate systems (well in excess of the desired 500 minimum) and the fact that computation using empirical distributions has not been demonstrated in previous criterion function research.

When continuous c.d.f. approximations are employed it is necessary to determine minima and maxima for the x_i (and therefore for the y_k and z_j as well) to facilitate curve fitting. With the use of empirical distributions this is no longer necessary since computation is accomplished via Eq. 4.46 (Parzen, 1960: 25) based on the observed step function (Feller, 1966: 36). However, it is recognized that additional insight may be gained via these determinations. Therefore, the minima and maxima for this demonstration are presented in Tables 22 (y_k), 23 (z_j), and 24 (x_i).

The next step called for in the design morphology is the Analysis of the Parameter (or Design) Space. In an actual implementation where design decisions will continuously be modified during the life of the

TABLE 22.

PARAMETER RANGES

	Parameters	Minimum	Maximum
1	Gross acquisition cost	0	165
2	Maturity of technology	0	2
3	Battery contractor successes	1	8
4	Battery contractor attempts	1	8
5	Fuel cell contractor successes	1	8
6	Fuel cell contractor attempts	1	8
7	Motor contractor successes	1	8
8	Motor contractor attempts	1	8
9	Cooling system contractor successes	1	8
10	Cooling system contractor attempts	1	8
11	Number of missions planned	300	300
12	APU operating hours per mission	1.5	1.5
13	System life expectancy	1.275	17,250
14	Cost of new/refurbished unit	40	19,800
15	Serial hours for replacement	6.8	27.6
16	Parallel hours for replacement	6.8	46
17	Cost of serial hours	28.7	28.7
18	Cost of parallel hours	19	19
19	Time between servicing	1.5	1.5
20	Quantity of consumables for servicing	0	325
21	Cost of consumables	10	50
22	Serial hours for servicing	6.8	52.9
23	Parallel hours for servicing	26.35	104.48
24	Disposal/retirement costs	0	10
25	Savings due to future applications/ext	0	34.5
26	Weight	624.5	5638.13
27	Volume	38.25	66
28	Operating temperature range	80	120
29	Crew operating requirements	10	15
30	Complexity	1	5
31	Impact on performance of other systems	1	3
32	Ability to facilitate beneficial changes	1	3
33	Acquisition time	0	5.75
34	Installation time	0	17
35	Mission length	1.5	1.5
36	Time between missions	6	6
37	Number of APU systems installed	3	3
38	Number of APU systems for mission completion	2	2
39	Tolerance to abuse	1	3
40	Potential severity of failure	1	3
41	Hazardous handling requirements	1	5

TABLE 23.

SUBMODEL RANGES

	Submodels	Minimum	Maximum
z_{11}	Expected initial cost	0	1320
z_{111}	Initial cost estimate	0	165
z_{112}	Risk factor	1	8
z_{1121}	Contractor risk	25	1
z_{12}	Recurring costs	2[illegible].75	8632.16
z_{121}	Replacement costs	.0095	7576.28
z_{1211}	# replacements required	.026	352.94
z_{1212}	Unit cost to replace	364.36	21,466.12
z_{122}	Servicing costs	208.74	1055.88
z_{1221}	# svcing actions reqd	300	300
z_{1222}	Cost of svcing action	695.81	3519.6
z_{13}	Long term costs	-34.5	10
z_{21}	Physical requirements	7.8×10^{7}	1.48×10^{11}
z_{22}	Impact on operations	10	45
z_{23}	Integration effects	.33	3
z_{31}	Initial availability	1	49.62
z_{32}	Ops availability	.9723	.9948
z_{321}	Total possessed hrs	4321.5	4321.5
z_{322}	Down time hours	22.58	119.74
z_{3221}	Maint down time	22.58	55.02
z_{32211}	Time to replace	13.6	73.6
z_{32212}	Time to service	33.15	157.38
z_{3222}	Mean time between maint	.6892	1.5
z_{33}	Prob mission completion	.1327	.9999
z_{41}	Flight safety	.0442	2.9997
z_{42}	Ground safety	.00139	.19929

TABLE 24.

CRITERION RANGES

	Criterion	*Minimum*	*Maximum*
x_1	Cost	.0001004	.0057389
x_2	Performance	5.005×10^{-14}	3.885×10^{-9}
x_3	Availability	.0026	.9947
x_4	Safety	.0000614	.5978102

project based on test results and other new information, this step provides invaluable feedback concerning how adjustments in design parameters should be made. When accomplished thoroughly, the activities of sensitivity and stability analysis can be expected to require large-scale programming and computational effort. If NASA ultimately decides to pursue the use of the morphology to assist their design process, this step would become essential. However, such efforts were judged to exceed the scope of the present demonstration since this research proposes no changes in the method of their accomplishment. Therefore, the description proceeds with formal optimization.

Formal Optimization

All computational work for this demonstration was accomplished on personal computers. In addition to facilitating the convenience of the author, avoidance of mainframe computing was desired to support the contention that the methods proposed can be easily implemented by prospective users, even in the absence of sophisticated hardware. The vast majority of computing was accomplished on the author's Zenith Z-171 portable. For the longest running step (conversion of raw criteria scores to X_i), a Compaq Model II portable with an 80286 processor was used to reduce the required run time.

Three separate programs were written to evaluate the defined set of candidate systems in accordance with the proposed method described

in Chapter 4. The programs were initially written and tested using interpreted BASIC. They were subsequently compiled and executed using TURBO BASIC in order to improve run times and provide sufficiently large variable storage capabilities. The process flow showing the relationship among the programs is depicted in Figure 51.

The first program, LITTLE-X.BAS, converted candidate system parameter values to raw criteria scores using the submodels previously described in this chapter. A flowchart of the program is shown in Figure 52 and a complete source code listing is contained in Appendix B. Given the large number of candidate systems, parameter values were initially stored in six data files. The program simply read in the parameter values for a candidate system, computed the submodels in Eq. 5.8-5.38 and stored the resulting raw criteria scores in an output file. This process was repeated until scores were computed for all candidate systems. The large number of candidate systems required the use of two such output files.

The second program, BIG-X.BAS, converted the raw criteria scores (x_i) produced by LITTLE-X.BAS to normalized scores (X_i), including intersection terms (X_{ij}). As mentioned previously, this was accomplished via a simple counting algorithm based on Eq. 4.46 and the empirical distribution of raw criteria scores. A flowchart of the program is given in Figure 53 and the program source listing is included as Appendix C. Since this program required the longest run time, two versions were run in parallel, each performing computations

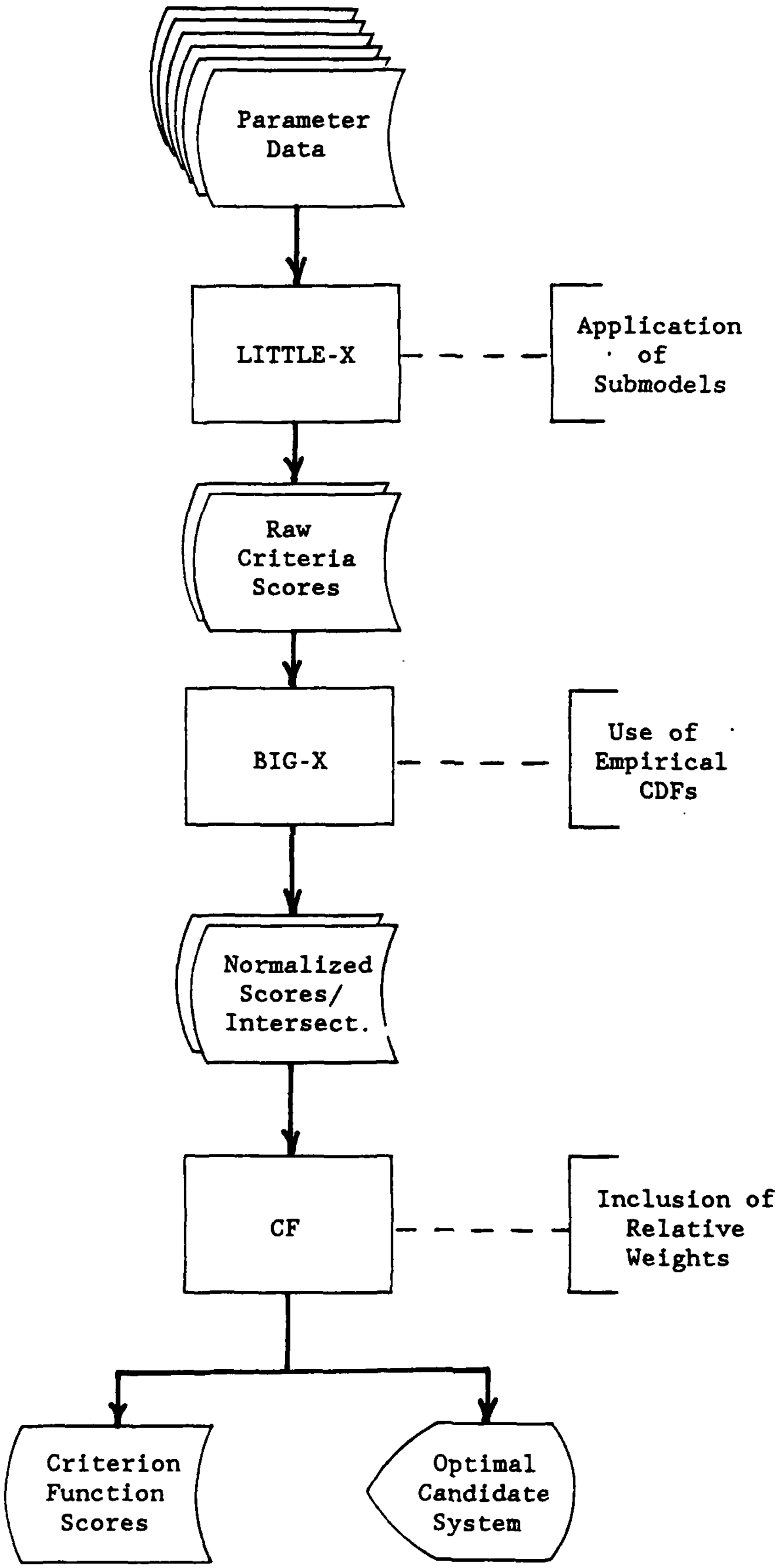


Figure 51. Overall Computation Flowchart

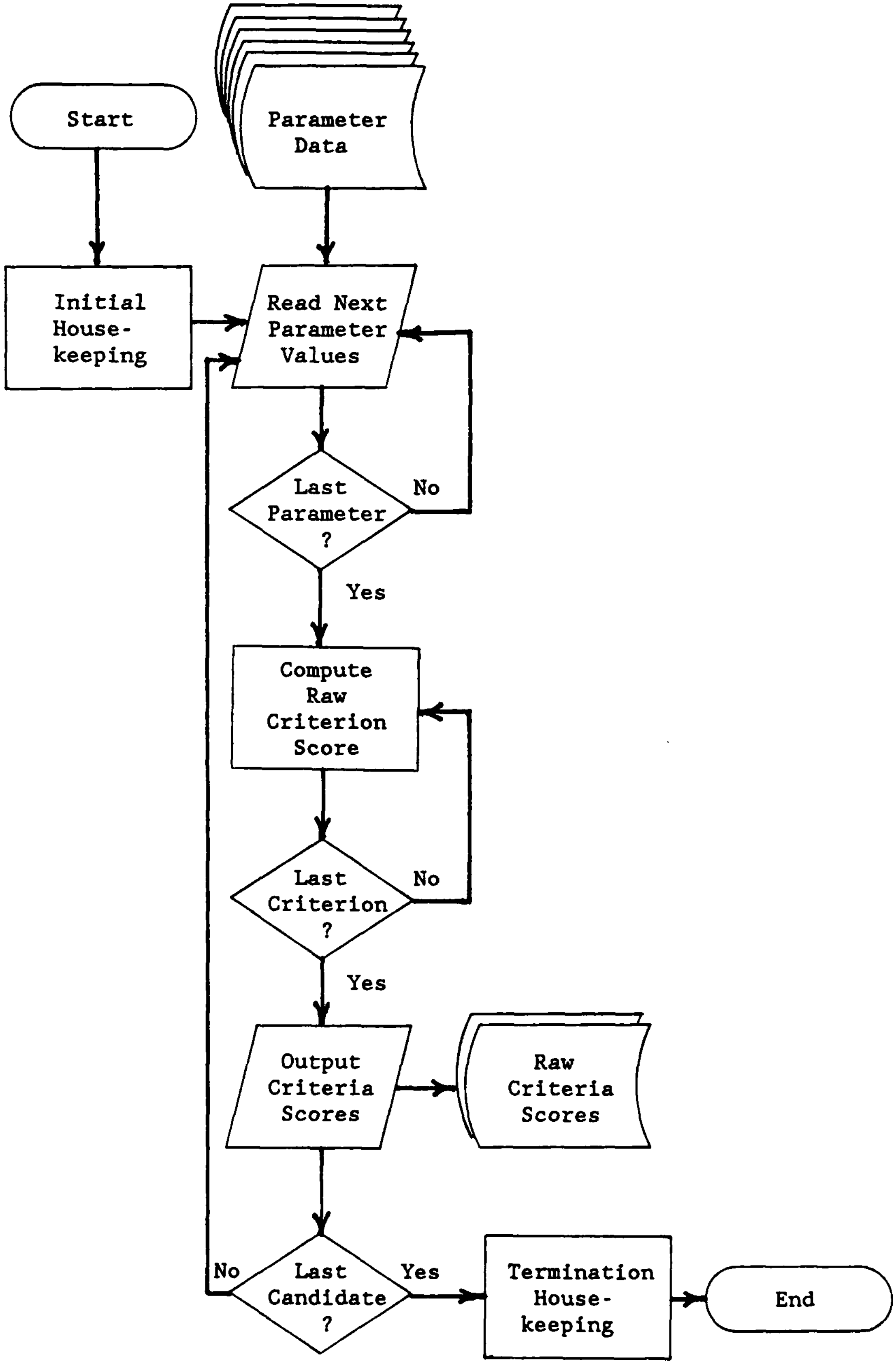


Figure 52. Computation of Raw Criteria Scores

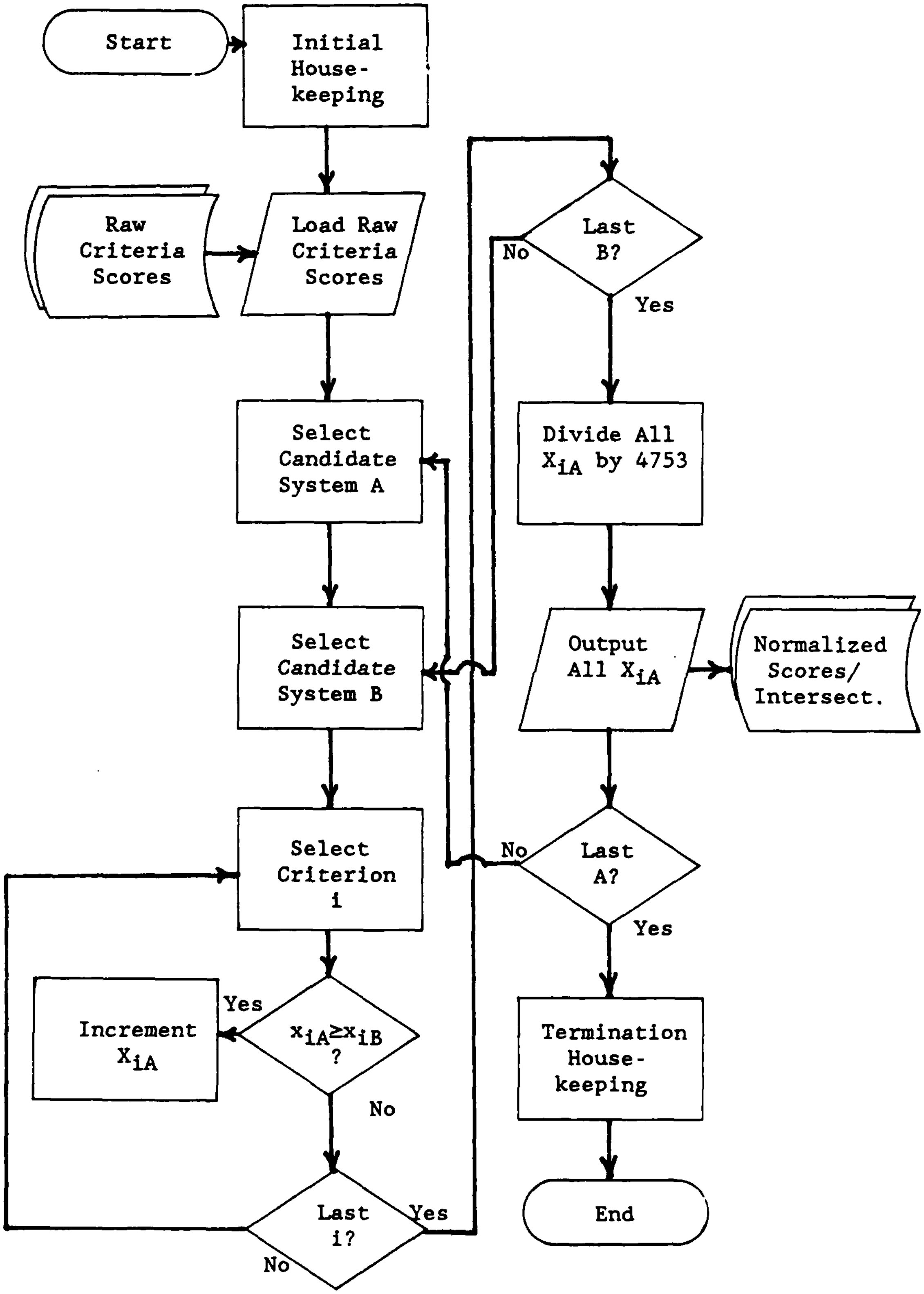


Figure 53. Conversion of Raw Criteria Scores

to Normalized Scores and Intersections

for half of the candidate systems, but both necessarily based on the distribution of the complete set of candidates. Each program produced one output file.

The final program, CF.BAS, read each candidate's X_i values (as output from BIG-X.BAS), determined the appropriate relative weights, normalized the weights, and wrote the resulting CF value to a single output file. The program also output the best and worst candidate systems to the screen along with their CF values. The logical flow of this program is shown in Figure 54 and the source listing is contained in Appendix D.

It should be noted that none of the program source code is particularly complex or lengthy. Furthermore, the programs may be easily adapted to solution of other four-criteria problems by simply changing:

a. The input and output files used by each program,

b. The number of candidate systems to be evaluated in each program,

c. The submodels in LITTLE-X.BAS, and

d. The relative weights in CF.BAS.

Only slightly more involved modifications would be required to adapt the programs to cases of other than four criteria. Even these modifications would be primarily programmatic in nature (more or fewer variables and loops) with the logical flow remaining unchanged.

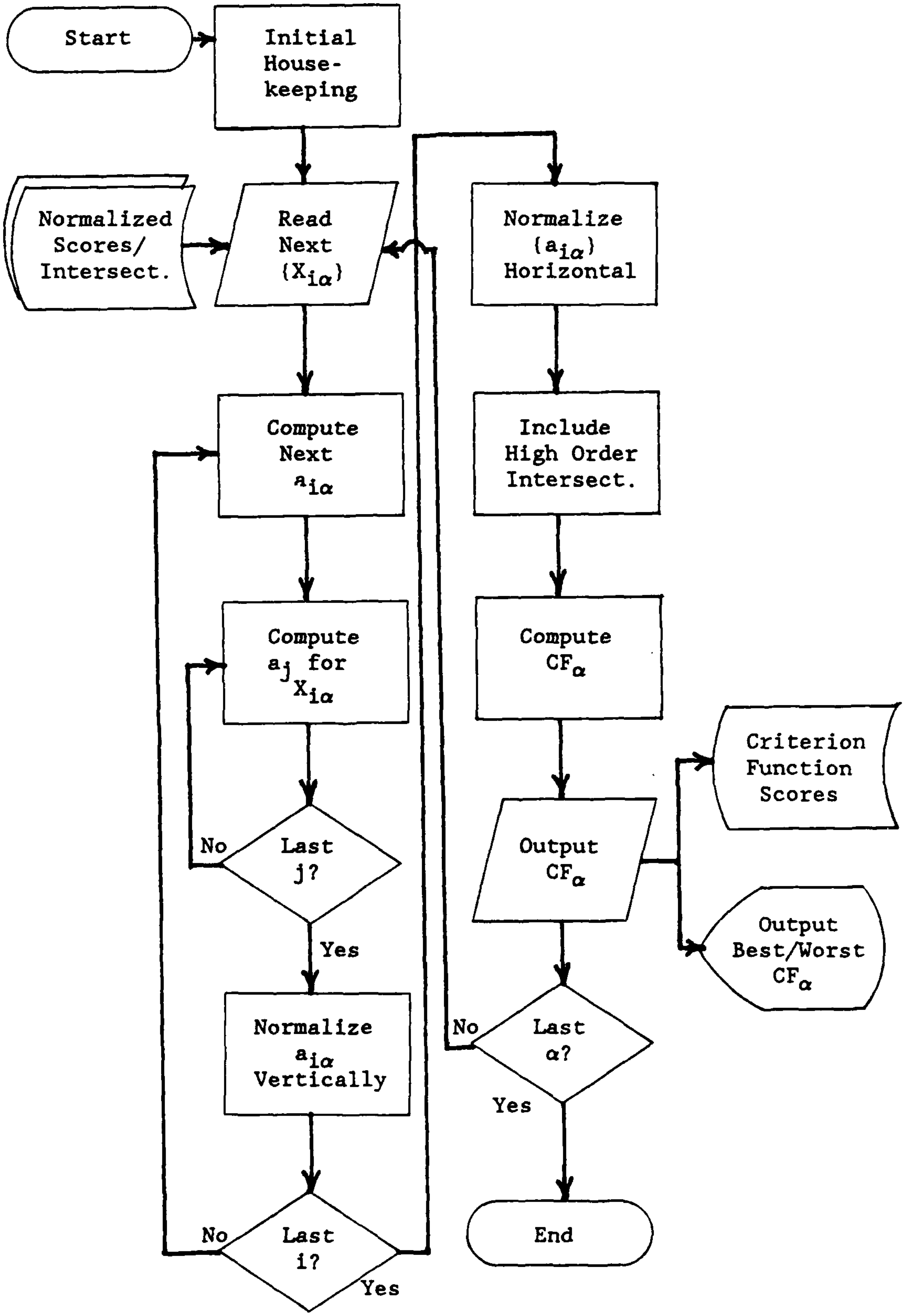


Figure 54. Computation of CF Values

This section has described the mechanics of the optimization process. The final section of this chapter reviews and discusses the specific results obtained for this demonstration.

Discussion of Results

Results described in this section were obtained using MINITAB, LOTUS 1-2-3, and a variety of ad hoc BASIC programs run against the various program output files.

Candidate system CF values ranged from a high of .9326 to a low of .3119. The optimal candidate system (CF=.9326) consisted of $Ag\text{-}H_2$ IPV batteries with a neutral design profile as the power source, a new independent cooling system (with a management-oriented design), switched-reluctance motors (also with a management-oriented design), and variable pressure pumps. The parameter values for the optimal candidate system are shown in Table 25. This system achieved X_1 scores greater than .75 on all four marginal criteria and greater than .5 on all intersection terms. Interestingly, it was also the best of all candidate systems with respect to criterion X_4, safety.

At this point in the analysis, it is appropriate to consider the effect of the accuracy of the results on the choice of the optimal candidate. Specifically, accuracy limitations on data estimates, computations, and curve fitting may all serve to moderate apparent

TABLE 25

OPTIMAL CANDIDATE PARAMETER VALUES

	Parameter	Value
1	Gross acquisition cost	138.53
2	Maturity of technology	1
3	Battery contractor successes	3
4	Battery contractor attempts	4
5	Fuel cell contractor successes	1
6	Fuel cell contractor attempts	1
7	Motor contractor successes	8
8	Motor contractor attempts	8
9	Cooling system contractor successes	8
10	Cooling system contractor attempts	8
11	Number of missions planned	300
12	APU operating hours per mission	1.5
13	System life expectancy	150
14	Cost of new/refurbished unit	108.24
15	Serial hours for replacement	8
16	Parallel hours for replacement	8
17	Cost of serial hours	28.7
18	Cost of parallel hours	19
19	Time between servicing	1.5
20	Quantity of consumables for servicing	0
21	Cost of consumables	0
22	Serial hours for servicing	8
23	Parallel hours for servicing	31
24	Disposal/retirement costs	0
25	Savings due to future applications/ext	10
26	Weight	1626.37
27	Volume	50
28	Operating temperature range	95
29	Crew operating requirements	10
30	Complexity	1
31	Impact on performance of other systems	1
32	Ability to facilitate beneficial changes	3
33	Acquisition time	3.2
34	Installation time	17
35	Mission length	1.5
36	Time between missions	6
37	# of APU systems installed	3
38	# of APU systems for mission completion	2
39	Tolerance to abuse	3
40	Potential severity of failure	1
41	Hazardous handling requirements	3

differences among CF scores. In other words, a small difference in CF scores may, in fact, be insignificant given the level of accuracy of the inputs to those results. In this demonstration, the accuracy of the original sources of estimation was generally unknown. Therefore, to facilitate comparison, it was assumed that the accuracy of the estimates was adequate to support all subsequent computations. In practice, perceptions of distinctions among candidates should not be based on differences which are finer than the accuracy of the constituent sources of measurement. Instead, candidates whose differences are not greater than the accuracy of their measurement should be treated as indistinguishable. In such cases, it may be most useful to pay more attention to groups of candidates formed by significant breaks in the ranking of CF scores.

The worst candidate system (CF=.3119) consisted of Li-$SOCl_2$ batteries, the existing freon cooling system, samarium cobalt motors (all with technically-oriented design profiles), and constant pressure pumps. This candidate's highest X_i score (X_2) was .4551 and its remaining X_i scores were all less than .1.

The current APU system achieved a CF score of .4828. A review of its scores indicated that the only reason it was able to obtain a score as high as it did is that it achieved the top rating in availability (X_4). This is explained by the fact that its <u>initial</u> availability (the system is already in use) overwhelmed the remaining candidate systems, all of which would require approximately two years

or more to implement. With the exception of X_4, the current system was able to achieve no higher than .2729 for any criterion.

Recognizing that the results for any single candidate system may be anomalous, Table 26 is presented to help support generalizations. To facilitate representation in the available space, the candidate systems are coded in the following manner:

1st position - Battery/fuel cell energy source
2nd position - Energy source design profile
3rd position - Cooling system (for battery-powered systems)
4th position - Cooling system design profile
5th position - Motor type
6th position - Motor design profile
7th position - Hydraulic pump option

The values used to represent alternatives in the first, third, and fifth positions were assigned sequentially in the order the alternatives were presented earlier in this chapter. The management-oriented, technically-oriented, and neutral design profiles are denoted as 1, 2, and 3, respectively. A "1" in the seventh position would indicate constant pressure pumps while a "2" indicates variable pressure pumps.

Examination of Table 26 suggests the following insights. First, the dominant single characteristic is the preference for variable pressure hydraulic pumps: it is the sole characteristic common to all 50 top performers. The second most clearly evident preference concerns the use of $Ag\text{-}H_2$ IPV batteries as the APU energy source. This characteristic is common to 47 of the top 50 candidates, with the next best energy source (Ag-Zn secondary batteries) not emerging until

TABLE 26

TOP 50 CANDIDATE SYSTEMS

Rank	Candidate	CF
1	7341312	0.9326
2	7341332	0.9314
3	7343312	0.9303
4	7321312	0.9287
5	7321332	0.9271
6	7343332	0.9270
7	7323312	0.9262
8	7323332	0.9238
9	7341112	0.9068
10	7341322	0.9063
11	7341132	0.9060
12	7343112	0.9049
13	7141332	0.9042
14	7143312	0.9038
15	7141312	0.9031
16	7321322	0.9030
17	7143332	0.9027
18	7341212	0.9025
19	7343132	0.9024
20	7321112	0.9013
21	7343212	0.9005
22	7241312	0.9004
23	7341232	0.9004
24	7321132	0.8999
25	7323112	0.8989
26	7321212	0.8988
27	7121332	0.8982
28	7342312	0.8981
29	7123312	0.8980
30	7121312	0.8972
31	7123332	0.8969
32	7221312	0.8968
33	7321232	0.8968
34	7343232	0.8966
35	7323212	0.8964
36	7323132	0.8964
37	7322312	0.8963
38	7311312	0.8955
39	7323232	0.8926
40	7311332	0.8919
41	7241332	0.8905
42	7313312	0.8905
43	7141322	0.8901
44	7142312	0.8870
45	4341312	0.8868
46	4341332	0.8867
47	7243312	0.8865
48	7221332	0.8860
49	7343322	0.8855
50	4343312	0.8855

NOTE: The top-rated fuel cell candidate was: 2100312, CF = .7953

the 45th position. As the note at the bottom of the table indicates, the top-rated fuel cell-powered candidate (using solid polymer electrolyte) failed to make the top 50, but scored a respectable .7953.

Perhaps the next most prominent observation emerging from the table is an apparent, although less pronounced, preference for the use of switched-reluctance motors. In fact, the most distinct break in the table's CF scores occurs between the eight (CF=.9238) and ninth (CF=.9068) candidates when switched-reluctance motors are first replaced with an alternative motor. Prior to that point in the rankings, the combination of Ag-H_2 IPV batteries, switched-reluctance motors, and variable pressure pumps were the only three characteristics common to all eight candidates.

Finally, it is apparent that management-oriented and neutral design profiles were generally preferred to technically-oriented profiles. This would seem to support the choice of more proven, conservative subsystem designs over state-of-the-art but riskier alternatives. This apparent preference may perhaps reflect the fact that although the decision makers participating in this demonstration are highly qualified in terms of technical knowledge, their duties and responsibilities might be described as primarily managerial in nature. Other possible explanations might include a general risk-averse orientation on the part of either the individual decision makers or the organization as a whole.

Chapter 6

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

The topic of this dissertation has been decision making with multiple, potentially interacting criteria and varying relative weights within the context of system design. The term "system" has been broadly defined as "any set of objects and/or attributes and the relationships among them." The term "design" has been used synonymously with "planning" and has been used to refer to a series of purposeful activities directed toward the achievement of a particular system.

Chapter 1 presented a review of significant developments in system design. The primary developments cited were the formalization of distinguishable design phases, explicit consideration of the complete set of system life cycle phases, the significant influence of systems theory, and the iterative nature of design. Chief among the specific influences of systems theory were the focus on a macro (vs. micro) level of design, an expansionist (as opposed to reductionist) perspective, and a deductive (as opposed to inductive) approach to

problem solution. The remainder of Chapter 1 delineated desirable characteristics of methods for design decision making. This discussion provided the foundation for the research goals by citing the need for a prescriptive decision model for complex, multi-criteria design decision problems. Specific desirable characteristics cited included the explicit consideration of both objective and subjective factors, the accommodation of relative weights which vary throughout the range of criteria performance, the objective of optimizing (rather than simply satisficing) criteria, and the ability to accommodate interaction among criteria.

Chapter 2 described a specific design/planning methodology which served as a framework for the balance of the research. In the Feasibility Study, the problem is generally defined and then further detailed for each of the system life cycle phases. Alternative solutions (candidate systems) are then synthesized and subsequently screened to ensure their suitability for further analysis. In the Preliminary Activities, broad decision criteria are defined and relative weights assigned. Each criterion is then described in terms of constituent elements which can be either directly measured (parameters) or modeled (submodels). The parameters are then related to the criteria via the submodels and the resulting criteria are combined with their relative weights in the form of the criterion function (CF). Following additional analysis, an optimal candidate system (selected based on CF values) is identified, tested further, and implemented based on plans formulated in the Detail Activities.

Chapter 3 focused on the issue of identifying the optimal candidate system by reviewing some of the foremost general and design-based quantitative decision making approaches which have been used or suggested within the context of system design. General approaches discussed included linear programming, goal programming, decision analysis, and dimensionless analysis. The two design-based approaches reviewed were the Tricotyledon Theory and the criterion function. Although each of the approaches have demonstrated merits, the criterion function was selected as the best starting point to achieve the desired characteristics stated in Chapter 1.

Chapter 4 began with a more detailed description of the current method of criterion function implementation. The description focused on the use of probability theory in the evaluation of criteria, the measurement of interactions, and the inclusion of relative weights. The chapter then continued with an examination of the logical basis for the form of the criterion function. Based on this discussion, the current methods were reviewed and sources of inconsistency identified. Specific revisions were proposed which resolve the inconsistencies in accordance with probability theory. In particular, the revisions drew on the concept of criteria intersections, which may be affected by the presence of interaction. Finally, a discussion was presented concerning the use of empirical distributions as an alternative to continuous approximations.

In Chapter 5, a demonstration of the revised approach was accomplished using the design of auxiliary power units for NASA's space shuttle. The demonstration was presented using the format of the framework design methodology. Four decision criteria (and their intersections) with continuously varying relative weights were used to evaluate a set of 4,753 identified candidate systems. The proposed criterion function method was implemented using a series of BASIC programs run on personal computers. The optimal candidate system was identified and generalizations were suggested based on a review of the fifty top-rated systems.

Conclusions

At the conclusion of Chapter 1, the research objective was defined to be <u>the development of a prescriptive design decision making approach</u> which:

1) supports comparative evaluation of a large number of design alternatives,

2) considers multiple, potentially conflicting objectives,

3) considers both objective and subjective relevant factors,

4) permits relative weights which vary both among criteria and within the range of possible criteria values,

5) seeks decisions which, within the context of the defined problem, are optimal in the sense suggested by Keen (1977) and discussed earlier in Chapter 1, and

6) considers interaction among criteria.

As a result of the discussion in Chapter 3, the criterion function was selected as the most promising basis for the achievement of the research objective. However, it was acknowledged that the current method of criterion function implementation was capable of producing results which are inconsistent with its basis in probability theory. It was therefore observed that a key goal for the success of the research would be the "clarification of the criterion function implementation technique to produce figures of merit on the range 0 to 1.0, consistent with the underlying theory."

In order to achieve this goal, Chapter 4 identified specific inconsistencies in previous CF implementation methods, identified the sources of these inconsistencies, and proposed revisions which resolved them. Chapter 5 then demonstrated the revised method on an actual design problem.

During the accomplishment of these activities, the following conclusions were obtained which are of primary importance with respect to the research objective.

1. Criteria intersections were meaningfully distinguished from criteria interaction.

2. Computation of criteria intersections (X_{ij}) required the specified revisions to achieve consistency with probability theory.

3. Computation of criteria relative weights (A_i, A_{ij}, etc.) required the specified revisions to achieve consistency with probability theory.

4. The resulting revised approach to criterion function implementation satisfies the stated research goal.

In addition, the following conclusions of secondary importance emerged from the activities of Chapters 4 and 5.

5. Additional insight was gained into the logical basis of the criterion function, especially with respect to the meaning of criteria intersection terms and intersection relative weights.

6. Empirical distribution functions were demonstrated as a useful alternative to continuous approximations in the evaluation of candidate system performance.

7. The problem of NASA's space shuttle APU design provided insight both in terms of the applicability of a theoretical approach to decision support and in terms of preliminary results based on data currently available.

8. The proposed methods were demonstrated using only simple BASIC

programs and personal computers, thereby supporting ease of implementation by practitioners.

Any research is subject to limitations. In addition to the specific assumptions mentioned at the outset of Chapter 5, the following limitations (all previously identified) are acknowledged.

1. The preliminary and, therefore, tentative nature of the data and estimates used for the demonstration in Chapter 5.

2. The depth and sophistication of the criteria modeling activities.

3. The lack of resources necessary to accomplish thorough sensitivity analysis.

4. The difficulty of empirically validating the superiority or optimality of the proposed method.

The first three limitations pertain to the demonstration and were addressed in Chapter 5. With respect to the fourth limitation (empirical validation), it was acknowledged in Chapter 1 that establishing the optimality of multi-criteria decision techniques is inherently problematic. It is precisely the issue of defining the "best" solution which differentiates alternative approaches. Direct comparison of the solutions produced by various methods is therefore impaired by the differences in the assumptions employed in each

approach. It was for this reason that we emphasized consistency with the underlying theory as the most important guide. However, empirical illustrations have been used where possible to compare the proposed revisions with previous implementation methods.

Recommendations for Future Research

Given the breadth of issues discussed and the flexibility of the methods presented here, numerous opportunities for future research are apparent. Those described in the following paragraphs would seem to represent particularly important areas.

First, a useful aid to researchers in this area would be empirical studies concerning the design methodologies currently used in practice and the success with which they support structured decision making. While numerous citations were presented in this research concerning various methodologies and their use, virtually all of these works are biased in the sense that their purpose is primarily to describe or promote a particular approach. There is an apparent lack of descriptive field work (e.g. surveys) which would provide insight concerning unaddressed needs of practitioners.

A second research need, also empirical in nature, is a method to support empirical validation or "fair" comparison of multi-criteria decision techniques. Direct comparison of the results produced by alternative methods will typically reveal simply different answers

attributable to different assumptions. While descriptive techniques may be validated using direct interaction with decision makers, a basis is needed to facilitate empirical comparison of prescriptive methods without sacrificing the treatment of subjective considerations.

Third, it would seem appropriate to pursue work promoting the value of the criterion function based on the merit of its explicit consideration of intersection relative weights. The capability to simultaneously trade off both marginal criteria performance <u>and</u> various combinations of balance is a unique strength of the method which would appear to be only vaguely approached by goal programming.

As mentioned in Chapter 4, the normalization scheme employed for relative weights is a very effective, yet subtle means of insuring consistency in the evaluation process. Further work is merited to fully describe and explore its significance both in the context of the criterion function and in other decision making techniques.

Another suggestion is the explicit consideration of relative weight tradeoffs as a supplement to the currently proposed sensitivity analysis. It is quite likely that in some applications, CF results may be highly responsive to specific changes in relative weights, thereby focusing the attention of decision makers on the most relevant aspects of the evaluation. By contrast, other implementation results

may prove quite insensitive to modifications of relative weights, thereby increasing the confidence of the decision makers.

Another issue is the apparent goal of CF (as described in Chapter 4) to seek that candidate system which best avoids a bad choice while many other multi-criteria approaches pursue proximity to the theoretic ideal. (These two alternative approaches were distinguished in Chapter 4 by the nature of their isopreference lines.) What are the tradeoffs between these two general approaches and do cases exist where one or the other is particularly appropriate?

A closely related issue is raised by the possible addition of a third dimension to the current matrix of eight CF models. The third dimension could be defined in terms of (at least) three discrete cases: A_{ij} are positive, negative, or zero. Again, isopreference curves may serve as a useful tool in comparing the appropriateness and effects of these three cases.

Finally, an obvious opportunity might be to pursue the use of the design morphology and the criterion function approach for NASA's APU and related design decisions (e.g. EMA). While the parameters, submodels, and even candidates considered would be expected to change in a more formal application, it was apparent that the criterion function responded directly to important aspects of the decision problem not normally accommodated by other approaches.

Appendix A

THEORETICAL BASIS FOR THE USE OF PROBABILITY IN CRITERIA MEASUREMENT

Note: The material in this appendix is essentially quoted directly from Ostrofsky (1977d: 332-345). It has been altered only to the small degree necessary to ensure grammatical and formatting appropriateness in its present context. The discussion begins with consideration of a single criterion, proceeds to the extension to two criteria, and concludes with the extension to multiple criteria in general.

Embedding a Criterion in a Probability Space

Define

γ - elementary event resulting from candidate system,

Ω - set of all elementary events γ; each event in Ω implies the occurrence of x,

x - design criterion with the range $x_{min} \leq x \leq x_{max}$,

Γ - Boolean field, i.e., a class of sets, C, such that if events A_1 and $A_2 \in C$, then A_1^c, $A_1 \cup A_2$, $A_1 \cap A_2$ are also in C.

Define a property of event A such that the event that A does not hold is the set of all elements in Ω not belonging to A, or, the complement of A, represented by A^c. It follows that the complement of Ω is the null set Φ.

From this notation an elementary event can be

$$X(\gamma_i) = x_i \qquad \text{Eq. A.1}$$

or

$$X^{-1}(x_i) = \gamma_i \qquad \text{Eq. A.2}$$

Equation A.1 simply states that each occurrence of the random variable implies a particular x_i and similarly Equation A.2 states that each occurrence of an x_i implies a particular γ_i; this notation can be read as "the preimage of x_i is γ_i." Hence, the elementary event γ_i occurs if an only if the respective value of the criterion, x_i, occurs since γ_i has been defined on the space of x_i such that a unique x_i exists for each γ_i.

Further, γ_i can be defined as the occurrence of a given set (or subset) of x_i such that the random event $X(\gamma_i)$ implies $\{x_i\}$. Hence the random event can be the occurrence of a design concept instead of the candidate system.

Some additional comments on x are appropriate at this point. In the context of the design morphology, x is considered the resulting

performance of a candidate system for a given concept. Since x is discrete, it should be the optimal value of the criterion which can be obtained from that particular candidate system. This is an important limitation on the definition of "candidate system" since criterion performance other than optimal can be obtained, and these alternatives would then define additional candidates for a given concept. The importance of this limitation becomes more evident when multiple criteria are considered and leads to the important property of being able to identify the performance of the total population of candidates in Ω.

The Probability Set Function

For each $A \in \Gamma$, a value $P(A)$ can be assigned and is considered the probability of A. $P(A)$ is then a set function over the members of Γ. The following axioms are offered for the function P by Rao (1965):

Axiom 1. $P(A) \geq 0$, $A \in \Gamma$.

Axiom 2. Let $A_1, \ldots, A_k$ $(A \in \Gamma)$ be disjoint sets whose union is Ω. Thus any elementary event, A_i, has one and only one of k possible descriptions, $A_1, \ldots, A_k$. Then the relative frequencies of the events $A_1, \ldots, A_k$ must sum to 1. Thus

$$\bigcup_{i=1}^{\infty} A_i = \Omega,\ A_i \cap A_j = \Phi \text{ for all } i \neq j \text{ where } \sum_{i=1}^{k} P(A_i) = 1 \qquad \text{Eq. A.3}$$

Then a set function P defined for all sets of Γ and satisfying axioms 1 and 2 is called a probability measure. The proof of Equation A.3 is given by Rao (1965). It then becomes apparent that

$$P(UA_i) = \Sigma P(A_i) \qquad \text{Eq. A.4}$$

for any countable union of disjoint sets in Γ whose union also belongs to Γ. Further,

$$(UA_i)U(UA_i)^c = \Omega = (UA_i)^c UA_1 UA_2 \ldots \qquad \text{Eq. A.5}$$

and hence

$$P(UA_i)+P[(UA_i)^c] = 1 = P[(UA_i)^c]+P(A_1)+P(A_2)+ \ldots \qquad \text{Eq. A.6}$$

The Borel Field and Probability Space

Since a Boolean field need not contain all countable sequences of sets, the field which does contain all such unions is the Borel Field, β, sometimes called σ field (Loeve, 1963; Rao, 1965). Given a field Γ (or any collection of sets) there exists a minimal Borel field, $\beta(\Gamma)$ which contains Γ. Since arbitrary intersections of Borel fields are also Borel fields, the intersection of all Borel fields containing Γ is precisely the minimal Borel field, $\beta(\Gamma)$. Since a Borel field is also a Boolean field, Axioms 1 and 2 and Equations A.3-A.6 apply.

Then for $\beta(\Gamma)$ there exists a unique function P^* which is the probability measure on $\beta(\Gamma)$ for the set A. Rao (1965) defines P^* as follows: "Consider a set A in $\beta(\Gamma)$ and a collection of sets A_i in Γ such that

$$A \text{ is a subset of } \bigcup_{i=1}^{\infty} A_i$$

Then

$$P^*(A) = \inf A_i \Sigma P(A_i)."$$

The calculus of probability is based on the space Ω of elementary events γ; a Borel field, β, of sets in Ω; and a probability measure P on β. This triplet (Ω, β, P) is called a probability space. The calculus is as follows:

$$C_x = A\{\gamma_i | i=1,\ldots,k\} \qquad \text{Eq. A.7}$$

and

$$C_x = \{x_i | i=1,\ldots,k\} \qquad \text{Eq. A.8}$$

and from Equations A.1 and A.2

$$X(\gamma_i) = X(A_i) = x_i \qquad \text{Eq. A.9}$$

and

$$X^{-1}(x_i) \text{ is a subset of } \beta \text{ and } X(A_i) \text{ is a subset of } \beta \qquad \text{Eq. A.10}$$

Then

$$P(A_i) = P[X(\gamma_i)=x_i] \qquad \text{Eq. A.11}$$

If x is a design criterion with known range $x_{min} \le x \le x_{max}$, a random variable $X(\gamma)$ can be defined such that $X(\gamma) = x$ for a given candidate system. $X(\gamma)$ then is defined to be a criterion random variable whose probability density results from the set of candidate systems. Then the set of candidate systems which have been synthesized for the

design during the feasibility study implies a set of values x from which inferences can be made concerning the possible set of values C_x for the design, and the sample space, Ω, is the set of possible candidate systems for the design, and inferences concerning the various attributes of Ω can be made from C_x implied by the candidates which have been synthesized.

Then Γ is the class of all sets of x which result from the set of recognized candidate systems. If the set of recognized candidate systems is exhaustive for the admissable range of x, then the union of all sets of x is a Borel field, β. When the union of all candidate systems within the admissable range of x is precisely the intersection of all Borel fields containing Γ, the minimal Borel field, $\beta(\Gamma)$ is achieved, that is, the smallest class of candidate systems which contain all the admissable x.

When considering a single design criterion the elementary event γ is identically equal to the design criterion x. Further, the event A for the single criterion is equivalent to the event γ_i (see Equations A.9 and A.10). Then, for each $x \in \beta$ a measure P(x) can be assigned. Thus a set function P over the members of β is achieved and can be considered a probability set for x. Hence a design criterion x can be transformed into a probability space, (Ω, β, P), such that $P[X(\gamma)=x]$ is analogous to P(A). Then Rao's (1965) Axioms 1 and 2 apply:

1. $0 \leq P[X(\gamma)=x] \leq 1$.

2. $\bigcup_{i=1}^{\infty} x_i = \Omega$, $x_i \cap x_j = \Phi$ for all $i \neq j$ where $\sum_{i=1}^{\infty} P[X(\gamma_i) = x_i] = 1$.

Consequently the theorems and axioms of probability apply to $P[X(\gamma_i) = x_i]$ as well as the notions of probability densities and probability distributions.

Fitting the derived criterion distribution function to one of the standard distribution forms facilitates computation. Thus a brief summary of distribution properties is presented at this point in the discussion. These properties are well known, and their proofs are presented in most texts on probability theory. This discussion follows the presentation and proofs given by Rao (1965) and lists the more relevant properties.

1. The distribution function (d.f.) defined on R, the real line, is

$$F(x) = P\{\gamma: X(\gamma) < x\} = P[X^{-1}(-\infty, x)] \qquad \text{Eq. A.12}$$

Since $x \geq x_{min}$,

$$F(x) = P[X^{-1}(x_{min}, x)] \qquad \text{Eq. A.13}$$

2.

$$F(-\infty) = 0 \qquad \text{Eq. A.14}$$

$$F(\infty) = F(x_{max}) = 1 \qquad \text{Eq. A.15}$$

3. F(x) is nondecreasing:

$$P[X^{-1}(-\infty, x_2)] \geq P[X^{-1}(-\infty, x_1)] \qquad \text{Eq. A.16}$$

4. $F(x)$ is continuous at least from the left.

5. The criterion random variable on (Ω, β, P) induces a probability measure on (R, β_1), and this probability measure is completely specified by $F(x)$.

6. The criterion random variable $X(\gamma)$ defines a Borel field of sets β_x which are subsets of β; β_x is a sub-Borel field.

7. A function f which maps points of R^n, the n-dimensional space into R is said to be a Borel function if

$$S \in \beta \text{ where } f^{-1}(S) \in \beta^n \qquad \text{Eq. A.17}$$

where S is a set of points.

The Class of Subsets for Two Criteria

Let R^2 be defined as the set of all elementary events with elements γ; each event in R^2 implies the occurrence of (x_1,x_2).

x_1 - design criterion 1 with range $x_{1\ min} \leq x_1 \leq x_{1\ max}$.

x_2 - design criterion 2 with range $x_{2\ min} \leq x_2 \leq x_{2\ max}$.

A - an event which denotes the occurrence of any combination of elementary events (i.e. any combination of the joint occurrences of x_{1i} and x_{2i}).

Γ - Boolean field, i.e. a class of sets, C, such that if events A_1 and $A_2 \in C$, then A_1^c, $A_1 \cup A_2$, $A_1 \cap A_2$ are also in C.

Then the criterion random variable is defined,

$$X(\gamma_i) = X(\gamma_{1i},\gamma_{2i}) = [X(\gamma_{1i}),\ X(\gamma_{2i})] \qquad \text{Eq. A.18}$$

where

$$X = (X_1,X_2) = (x_1,x_2) \qquad \text{Eq. A.19}$$

so that usually

$$X^{-1}(x_1,x_2) = \gamma_i$$

Equation A.18 states that the elementary event γ_i is the joint occurrence of a value for each of the two subevents, γ_{1i} and γ_{2i}, and that the criterion random variable, X, is the joint occurrence of a value from each of the two design criteria. Then from Equation A.19 each occurrence of (x_{1i},x_{2i}) implies a particular γ_i, or the preimage of $(\gamma_{1i},\gamma_{2i})$ is γ_i. Then, in the context of design, (x_{1i},x_{2i}) represents the joint occurrence of a real number for each criterion, respectively, which results from a candidate system. The set of candidate systems which is exhaustive then provides the ability to define R^2.

It is recalled that each candidate system provides one number for (x_{1i},x_{2i}) so that variations in the performance of each x_{ki} are considered to yield different candidates. This restriction enhances the ability to discriminate among candidates which yield superior numbers on only one of the x's.

The Probability Set Function and Borel Field

Two single-valued real functions which map Ω into R^2, two-dimensional space, result in a two-dimensional criterion random variable. Parzen (1960) describes the probability function, P[.], as "a numerical 2-tuple-valued random phenomenon whose value, P(A) at any set A of 2-tuples of real numbers represents the probability that an observed occurrence of the random phenomenon will have a description lying in the set A." Parzen further identifies the probability, P[.], as analogous to a distribution of unit mass of some substance called probability over a two-dimensional plane which is calibrated with a coordinate system. Then for any set A of 2-tuples P(A) states the weight of the probability substance distributed over the set A.

Then Axioms 1 and 2 originally defined for one single-valued real function apply equally well to the two single-valued real functions when A and Γ are suitably redefined. Hence Equations A.4-A.7 apply for the two single-valued functions.

The Borel field, β^2, is then defined, as in the single variate case, as the field which contains all countable sequences of sets and their unions. Again, the minimal Borel field, $\beta^2(\Gamma)$, is defined from the intersection of all Borel fields containing precisely Γ.

Having defined the sample space in terms of R^2, the Borel field

as β^2, and the probability measure as P, the criterion probability space becomes (R^2, β^2, P).

Further,

$$A^1 = \{\gamma_1\}, \text{ a subset of } \Omega_1$$

Eq. A.20

$$A^2 = \{\gamma_2\}, \text{ a subset of } \Omega_2$$

and

$$A = A_1 * A_2, \text{ a subset of } R^2 = \Omega_1 * \Omega_2 \qquad \text{Eq. A.21}$$

Also, the preimage of the criterion random variable (X_1,X_2) is contained in the Borel field; hence the preimage of each candidate system is also in the Borel field:

$$X^{-1}(x_{1i},x_{2i}) \text{ is a subset of } \beta^2 \qquad \text{Eq. A.22}$$

Hence

$$P(A_i) = P[X(\gamma_i) = (x_{1i},x_{2i})] \qquad \text{Eq. A.23}$$

<u>Joint Probability Distribution and Conditional Distribution</u>

Let (X_1,X_2) be the two-criterion random variable having the probability space (R^2, β^2, P). Then Wilks (1962) defines E to be the interval $(-\infty,-\infty;X_1,X_2]$ in R^2 and

$$F(x_1,x_2) = P(E) = P[\gamma:X_1(\gamma)\leq x_{1i},X_2(\gamma)\leq x_{2i}] \qquad \text{Eq. A.24a}$$

which is clearly a single-valued, real, non-negative function of

(X_1,X_2) in R^2. Then any interval I of the form $(x_{1i},x_{2i};x_{1j},x_{2j}) \epsilon \beta^2$ since

$$I = [^{E}(x1j,x2j)^{-E}(x1i,x2j)^{-E}(x1j,x2i)^{-E}(x1i,x2i)] \quad \text{Eq. A.24b}$$

where i and j are two candidate systems. Then the probability that $(X_1,X_2) \epsilon I$ is

$$P[(X_1,X_2)\epsilon I] = F(x_{1i},x_{2i})-F(x_{1i},x_{2j})-F(x_{1j},x_{2i})-F(x_{1j},x_{2j})$$

Eq. A.25

Wilks calls the expression on the right of this equation the "second difference of $F(x_1,x_2)$ over I" and denotes this difference by

$$\delta_1{}^2F(x_1,x_2) \quad \text{Eq. A.26}$$

Figure 55 relates to the various quantities as follows:

$F(x_{1j},x_{2j})$ = doubly shaded area.

$F(x_{1i},x_{2j})$ = area shaded vertically.

$F(x_{1j},x_{2i})$ = area shaded horizontally.

$F(x_{1i},x_{2i})$ = area below and to the left of coordinates.

$\delta_1{}^2F(x_1,x_2)$ = unshaded area below and to the left of (x_{1j},x_{2j})

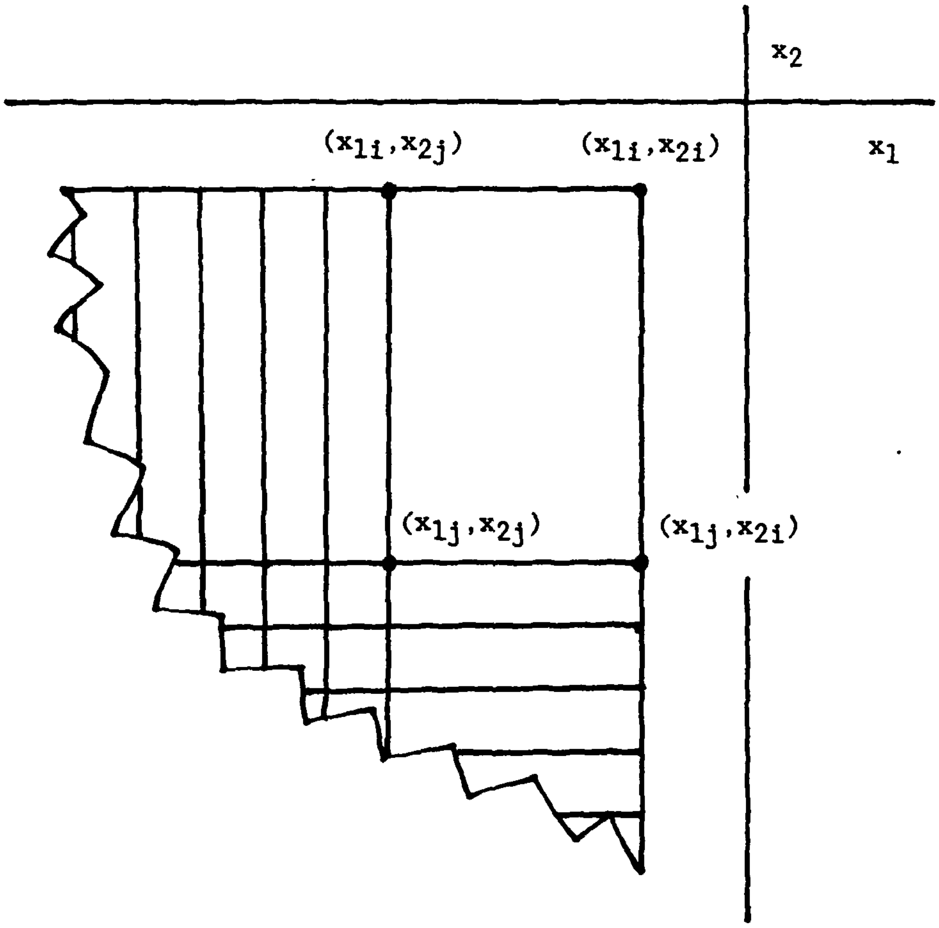


Figure 55. Second Difference of $F(x_1, x_2)$ Over I

(Ostrofsky, 1977d)

Further,

$$F(-\infty,x_2) = 0 \qquad \text{Eq. A.27}$$

$$F(x_1,-\infty) = 0 \qquad \text{Eq. A.28}$$

$$F(\infty,\infty) = 1 \qquad \text{Eq. A.29}$$

Also, if (X_1,X_2) is a criterion random variable with probability space (R^2, β^2, P), a unique function $F(x_1,x_2)$ exists defined by Equation A.24a at every point in R^2. Conversely, a function $F(x_1,x_2)$ defined by Equation A.24 at least continuous from the left and conforming with Equations A.26-A.29 uniquely determines a probability space (R^2, β^2, P). Further discussion of these two statements is given by Wilks (1962). Hence, $F(x_1,x_2)$ is called the distribution function of the two-dimensional random variable (X_1,X_2). $F(x_1,x_2)$ may also be referred to as a "bivariate criterion distribution function."

The marginal distributions are

$$F_1(x_{1i}) = F(x_{1i},\infty) = P[X_1(\gamma)\leq x_{1i}] \qquad \text{Eq. A.30}$$

$$F_2(x_{2i}) = F(\infty,x_{2i}) = P[X_2(\gamma)\leq x_{2i}] \qquad \text{Eq. A.31}$$

If one considers the total probability function as being distributed in the (X_1,X_2) plane in accordance with $F(x_1,x_2)$, then $F_1(x_1)$ is the projection on the X_1 axis and $F_2(x_2)$ is the projection on the X_2 axis. Hence it follows that $F_1(x_1)$ and $F_2(x_2)$ determine the probability spaces

$$[R_1{}^{(1)},\beta_1{}^{(1)},P^{(1)}] \text{ and } [R_2{}^{(1)},\beta_2{}^{(1)},P^{(2)}]$$

respectively, where $R_1^{(1)}$ and $R_2^{(1)}$ are the sample spaces denoted by Ω in the one-dimensional case.

When (X_1,X_2) is a bivariate, criterion random variable, having $F(x_1,x_2)$, Wilks (1962: 43) shows that a necessary and sufficient condition for X_1 and X_2 to be independent is that

$$F(x_1,x_2) = F(x_1)*F(x_2) \qquad \text{Eq A.32}$$

Further, let I be the set in R^2 for which $x_{1i}<x_1\leq x_{1j}$ such that $P(I) > 0$; then

$$F(x_1,x_2|I) = \frac{P(E\ I)}{P(I)} \qquad \text{Eq. A.33}$$

where E is defined as in Equation A.24a to be $(-\infty,-\infty,X_1,X_2]$. Hence the conditional probability of a bivariate, criterion random variable given any set in the sample space R^2 is the expression of Equation A.33.

When $F_1(x_1)$ is the marginal distribution function of x_1,

$$F(x_{1i},x_2|I) = \frac{F(x_{1j},x_2)-F(x_{1i},x_2)}{F_1(x_{1j})-F_1(x_{1i})} \qquad \text{Eq. A.34}$$

When the limit on the right side exists as x_{1i} approaches x_{1j} then it is denoted by

$$F(x_2|x_{1j}) = \lim_{x_{1i} \to x_{1j}} F(x_{1j}, x_2|I) \qquad \text{Eq. A.35}$$

If $F(x_2|x_{1j})$ exists for every value of X_2, then the $F(x_2|x_{1j})$ is a distribution function and has all the properties of a d.f. and is called the conditional distribution function of X_2 given $X_1 = x_{1j}$, or, when there is no possibility of ambiguity, $F(x_2|x_1)$, where X_1 is regarded as a parameter of given value, not as a random variable. Wilks (1962) identifies $x_2|x_1$ as a one-dimensional random variable whose distribution function $F(x_2|x_1)$ for a fixed value of X_1 is defined at all points in the X_1X_2 plane for which $X_1 = x_{1j}$ in such a way that $F(x_{2j}|x_{1j})$ is approximately the amount of probability (or probability density) lying along the portion of the line $X_1 = x_{1j}$ for which $X_2 \leq x_{2j}$.

Finally, for every set B of 2-tuples (Parzen, 1960)

$$P(B) = \iint_B f(x_1, x_2)\,dx_1 dx_2 \qquad \text{Eq. A.36}$$

where $f(x_1, x_2)$ is the criterion probability density function. Then the criterion distribution function becomes

$$F(x_1,x_2) = \int_{-\infty}^{x_{1i}} \int_{-\infty}^{x_{2i}} f(x_1,x_2)dx_1dx_2 \qquad \text{Eq. A.37}$$

and

$$f(x_1,x_2) = \frac{\delta^2}{\delta X_1 \delta X_2} F(x_1,x_2) \qquad \text{Eq. A.38}$$

where the second-order mixed partial derivative exists. Obviously Equation A.38 reduces to Equation A.23 when $X_1 = x_{1i}$ and $X_2 = x_{2i}$ so that

$$f(x_1,x_2) = f(x_{1i},x_{2i}) = P(A_i) \qquad \text{Eq. A.39}$$

The Class of Subsets for Multiple Criteria

Let R^n be defined as the set of all elementary events γ; each event in R^n implies the occurrence of $(x_1,...,x_n)$, where

x_k = kth design criterion with the range $x_{k\ min} \le x \le x_{k\ max}$, $k=1,...,n$.

A = an event which denotes the occurrence of any of elementary events.

Γ = Boolean field defined in a manner similar to that used previously.

If the real line $(-\infty,\infty)$ is represented by R and the n-dimensional real Euclidean space by R^n, then the points of R^n can be represented by the

set of real coordinates $(x_1,\ldots,x_n)$, where x_k is defined as shown above. Then the criterion random variable is defined for the ith candidate system as

$$X(\gamma_i) = (x_{1i},\ldots,x_{ki}) \qquad \text{Eq. A.40}$$

and

$$X^{-1}(x_{1i},\ldots,x_{ki}) = \gamma_i \qquad \text{Eq. A.41}$$

Thus for the design of a system with multiple criteria, each candidate system provides one real number for each x_{ki}, $k=1,\ldots,n$. As before, any variation in the system performance which leads to a different number, x_{ki}' to be considered as the design value, implies that the system must be considered as a different candidate. Consequently, as in the bivariate case, an enhanced ability to discriminate among candidates exists as well as the ability to define an increased number of candidates from which to choose.

The Probability Set Function and Borel Field

When n single-valued, real functions exist which map Ω in R^n the collection may be called an n-dimensional criterion random variable. As in the bivariate case Rao's (1965) Axioms 1 and 2 apply to the n-variate probability, and Equations A.4-A.7 apply.

Also, the Borel field, β^n, is again defined as the field which

contains all countable sequences of sets and their unions. Further, the minimal Borel field, $\beta^n(\Gamma)$, is defined from the intersection of all Borel fields containing precisely Γ.

Again, the probability space becomes (R^n, β^n, P), and Equations A.7-A.10 apply with the newly defined A_i, γ_i, and β^n, and Equation A.11 becomes

$$P(A_i) = P[X(\gamma_i) = (x_{1i},\ldots,x_{ki})] \qquad \text{Eq. A.42}$$

<u>Multicriterion Probability Distribution</u>

The previous results extend directly to the n-dimensional case. If $(X_1,\ldots,X_n)$ is an n criterion random variable whose probability space is (R^n, β^n, P) and $E(x_{1i},\ldots,x_{ki})$ is the set $(-\infty,\ldots,-\infty;X_1,\ldots,X_n)$ in R^n, then

$$F(x_1,\ldots,x_n) = P(E) = P[\gamma:X_k(\gamma)\leq x_{ki},k=1,\ldots,n] \qquad \text{Eq. A.43}$$

where i=ith candidate system. Thus, as in the two criterion case, $F(x_1,\ldots,x_k)$ is clearly a single-valued, real, and non-negative function of $(X_1,\ldots,X_k)$ in R^n.

Any interval $I \in R^n$ of the form $(x_{1i},\ldots,x_{ni};\ x_{1j},\ldots,x_{nj})$ belongs to R^n since

$$I = E_{(x1j,\ldots,xnj)} - [E_{(x1j,x2j,\ldots,xnj)} \cup \ldots \cup E_{(x1j,x2j,\ldots,xn-1,xki)}] \qquad \text{Eq. A.44}$$

Then the probability that $(X_1,\ldots,X_n) \in I$ can be found in terms of $F(x_1,\ldots,x_n)$ by

$$\begin{aligned} P[(X_1,\ldots,X_n) \in I] = F(&x_{1j},\ldots,x_{nj}) \\ &- [F(x_{1i},x_{2j},\ldots,x_{nj}) + \ldots \\ &\quad + F(x_{1j},\ldots,x_{k-1,j},x_{ni})] \\ &+ [F(x_{1i},x_{2i},x_{3j},\ldots,x_{nj}) + \ldots \\ &\quad + F(x_{1j},\ldots,x_{n-2,j},x_{n-1,i},x_{ni})] \\ &\ \vdots \\ &+ (-1)^n F(x_{1i},\ldots,x_{ni}) \end{aligned} \qquad \text{Eq. A.45}$$

Note: This notation is a modified form of that used by Wilks (1962); also see Parzen (1960). Then the right side of this equation can be denoted by $\delta_I{}^n F(x_1,\ldots,x_n)$ over I so that

$$P[(X_1,\ldots,X_n) \in I] = \delta_I{}^k F(x_1,\ldots,x_k) \geq 0 \qquad \text{Eq. A.46}$$

By argument similar to Equation A.27

$$F(-\infty,X_2,\ldots,X_n) = \ldots = F(X_1,X_2,\ldots,X_{n-1},-\infty) = 0 \qquad \text{Eq. A.47}$$

and

$$F(\infty,\ldots,\infty) = 1 \qquad \text{Eq. A.48}$$

Further,

$$F(x_1,\ldots,x_{i-1},x_{i+0},x_{i+1},\ldots,x_n) = F(x_1,\ldots,x_n) \quad i=1,\ldots,n$$

Eq. A.49

which states that the function is continuous on the right. Further, $F(x_1,\ldots,x_k)$ is unchanged if any set of x_k, say $(x_{1i},\ldots,x_{ni})$, is replaced by $(x_{1i+0},\ldots,x_{ni+0})$, respectively.

Hence if $(X_1,\ldots,X_n)$ is an n-dimensional criterion random vector with the probability space (R^n, β^n, P), there exists a function $F(x_1,\ldots,x_n)$ defined at every point in R^n by Equation A.43 which has the properties identified in Equations A.45-A.49.

Conversely, if $F(x_1,\ldots,x_n)$ is defined by Equation A.43 and has the properties of Equations A.45-A.49, the probability space (R^n, β^n, P) is obtained.

The function $F(x_1,\ldots,x_n)$ can be called the multicriterion distribution function and is directly analogous to the multivariate, cumulative distribution function of probability theory.

Marginal and Conditional Distributions

The marginal criterion distribution function of X_1, $F_1(x_1)$, is defined as

$$F_1(x_1) = F(x_1, \infty, \ldots, \infty) \qquad \text{Eq. A.50}$$

with the other marginal distributions being similarly defined:

$$F_i(x_i) = F(\infty, \ldots, x_i, \ldots, \infty) \qquad \text{Eq. A.51}$$

More generally, the marginal distribution function of $(X_1, \ldots, X_k)$, $k<n$, is defined by

$$F_{1,\ldots,k}(x_1, \ldots, x_k) = F(x_1, \ldots, x_k, \infty, \ldots, \infty) \qquad \text{Eq. A.52}$$

As in the two-dimensional case, suppose the random vector X to be distributed as $F(x_1, \ldots, x_n)$; then $F_1(x_1)$ is the projection on the x_1 axis, and, more generally, $F_i(x_i)$ is the projection on the ith axis of the n-dimensional hyperplane, each criterion having a one-dimensional probability space.

Wilks (1962) identifies the necessary and sufficient condition for $(X_1, \ldots, X_k)$ and $(X_{k+1}, \ldots, X_n)$ to be independent as

$$F(x_1, \ldots, x_n) = F(x_1, \ldots, x_k) * F(x_{k+1}, \ldots, x_n) \qquad \text{Eq. A.53}$$

where the two functions on the right are marginal distribution functions of the respective multicriterion random variable.

Also, $X_1,\ldots,X_n$ are mutually independent if and only if

$$F(x_1,\ldots,x_n) = F_1(x_1)*\ldots*F_n(x_n) \qquad \text{Eq. A.54}$$

Equation A.36 for n dimensions becomes

$$P(B) = \iint_B\ldots\int f(x_1,\ldots,x_n)dx_1,\ldots,dx_n \qquad \text{Eq. A.55}$$

and Equation A.37 becomes

$$F(x_1,\ldots,x_n) = \int_{-\infty}^{x_{ni}}\ldots\int_{-\infty}^{x_{1i}} f(x_1,\ldots,x_n)dx_1,\ldots,dx_n \qquad \text{Eq. A.56}$$

Also,

$$f(x_1,\ldots,x_n) = \frac{\delta^n}{\delta x_1,\ldots,\delta x_n}F(x_1,\ldots,x_n) \qquad \text{Eq. A.57}$$

Conditional distributions for the multicriterion case are extensions of the two-dimensional case. Let $X = (X_1,\ldots,X_n)$, the n-dimensional criterion random vector with distribution function $F(x_1,\ldots,x_n)$, and let $F_{1\ldots k}(x_1,\ldots,x_k)$ be the marginal distribution function of $(X_1,\ldots,X_k)$, $k<n$, and let I be the set in R^n for which $x_{mi}<x_m\leq x_{mj}$, $m=1,\ldots,k$; then the projection of this set on R^k, the sample space of $(X_1,\ldots,X_k)$, is the k-dimensional interval

$$I = (x_{1i},\ldots,x_{ki};x_{1j},\ldots,x_{kj}], \text{ a subset of } R^k \qquad \text{Eq. A.58}$$

Let $\delta_I{}^k F_{1\ldots k}(x_1,\ldots,x_k)$ be the kth difference of $F_{1\ldots k}(x1,\ldots,x_k)$ over I as defined in Equations A.45 and A.46. This kth difference is P(I) which is greater than 0. Now

$$F(x_1,\ldots,x_n|I) = \frac{P[E_{(x1,\ldots,xn)}\ I]}{P(I)} \qquad \text{Eq. A.59}$$

and

$$F(x_{1j},\ldots,x_{kj},x_{k+1},\ldots,x_n|I) = \frac{\delta_I{}^k F(x_1,\ldots,x_n)}{\delta_I{}^k F_{1\ldots k}(x_1,\ldots,x_k)} \qquad \text{Eq. A.60}$$

where the numerator is the kth difference of $F(x_1,\ldots,x_n)$ with respect to $(x_1,\ldots,x_k)$ for fixed values of $(x_{k+1},\ldots,x_n)$.

If the limit on the right-hand side of Equation A.59 exists as x_{1i} approaches x_{1j}, ... , as x_{ki} approaches x_{kj}, that is,

$$\lim_{x_{mi} \to x_{mj}} \{F(x_{1j},\ldots,x_{kj},x_{k+1},\ldots,x_m|I),\ m=1,\ldots,k\}$$
$$= F(x_{k+1},\ldots,x_n|x_{1j},\ldots,x_{nj}) \qquad \text{Eq. A.61}$$

then this is called the conditional random vector

$$(X_{k+1},\ldots,X_n|X_1,\ldots,X_k)$$

and has all the properties of an (n-k)-dimensional random vector.

When $(X_1,\ldots,X_n)$ is a continuous random vector, the conditional probability density is

$$f(x_{k+1},\ldots,x_n|x_1,\ldots,x_k) = \frac{f(x_1,\ldots,x_n)}{f_{1\ldots k}(x_1,\ldots,x_k)} \qquad \text{Eq. A.62}$$

Also,

$$\begin{aligned} f(x_1,\ldots,x_k) = {} & f(x_k|x_1,\ldots,x_{k-1}) \\ & * f_{1\ldots k-1}(x_{k-1}|x_1,\ldots,x_{k-2}) \\ & * \ldots \\ & * f_1(x_1) \end{aligned} \qquad \text{Eq. A.63}$$

assuming that the densities of all the conditional random variables exist.

Appendix B

SOURCE LISTING FOR LITTLE-X.BAS

```
10 REM *********************************************************
20 REM *********************************************************
30 REM
40 REM  LITTLE-X.BAS COMPUTES RAW CRITERION SCORES FOR
50 REM   CANDIDATE SYSTEMS BASED ON PARAMETER VALUES STORED
60 REM    IN SIX DATA FILES.  THE RESULTS ARE STORED IN
70 REM     DATA FILES B:LITTLEX1.DAT AND B:LITTLEX2.DAT.
80 REM
90 REM *********************************************************
100 REM ********************************************************
110 REM  THIS MODULE PERFORMS INITIAL HOUSEKEEPING
120 REM ********************************************************
130 PRINT TIME$
140 OPTION BASE 1
150 DEFINT F,I,J,K
160 MAX=4753
170 DIM Y(41)
180 DIM CAND(MAX)
190 DIM X1A(MAX)
200 DIM X2A(MAX)
210 DIM X3A(MAX)
220 DIM X4A(MAX)
230 DIM XMIN(4)
240 DIM XMAX(4)
250 FOR I=1 TO 4
260   XMIN=9999
270   XMAX=0
280 NEXT I
290 REM ********************************************************
300 REM  THIS MODULE READS IN PARAMATER VALUES FOR ALL CANDIDATE
          SYSTEMS
310 REM   AND INVOKES THE MODULE WHICH COMPUTES RAW CRITERION
          SCORES
320 REM ********************************************************
330 FIRST=1
340 LAST=793
350 FOR F=1 TO 6
360     IF F=1 THEN INPUT "PUT 1ST INPUT DISK IN B: AND PRESS
ENTER";NIL$
370   IF F=1 THEN OPEN "B:PARA-1.DAT" FOR INPUT AS #1
380   IF F=2 THEN OPEN "B:PARA-2.DAT" FOR INPUT AS #1
```

```
390    IF F=3 THEN INPUT "PUT 2ND INPUT DISK IN B: AND PRESS
ENTER";NIL$
400    IF F=3 THEN OPEN "B:PARA-3.DAT" FOR INPUT AS #1
410    IF F=4 THEN OPEN "B:PARA-4.DAT" FOR INPUT AS #1
420    IF F=5 THEN INPUT "PUT 3RD INPUT DISK IN B: AND PRESS
ENTER";NIL$
430    IF F=5 THEN OPEN "B:PARA-5.DAT" FOR INPUT AS #1
440    IF F=6 THEN OPEN "B:PARA-6.DAT" FOR INPUT AS #1
450    GOTO 750
460    CLOSE #1
470    FIRST=LAST+1
480    LAST=FIRST+791
490    PRINT F;" ";TIME$
500 NEXT F
510 REM ***********************************************************
520 REM  THIS MODULE OUTPUTS RAW CRITERION SCORES TO DATA FILES
530 REM ***********************************************************
540 INPUT "PUT 1ST OUTPUT DISK IN B: AND PRESS ENTER";NIL$
550 OPEN "B:LITTLEX1.DAT" FOR OUTPUT AS #2
560 FOR I=1 TO 2377
570   PRINT #2, CAND(I);X1A(I);X2A(I);X3A(I);X4A(I)
580 NEXT I
590 CLOSE #2
600 INPUT "PUT 2ND OUTPUT DISK IN B: AND PRESS ENTER";NIL$
610 OPEN "B:LITTLEX2.DAT" FOR OUTPUT AS #2
620 FOR I=2378 TO MAX
630   PRINT #2, CAND(I);X1A(I);X2A(I);X3A(I);X4A(I)
640 NEXT I
650 CLOSE #2
660 REM ***********************************************************
670 REM  THIS MODULE PERFORMS TERMINATION HOUSEKEEPING
680 REM ***********************************************************
690 LPRINT DATE$;" ";TIME$
700 FOR I=1 TO 4
710   LPRINT "XMIN(";I;")= ";XMIN(I);
720   LPRINT " XMAX(";I;")= ";XMAX(I)
730 NEXT I
740 END
750 REM ***********************************************************
760 REM  THIS MODULE COMPUTES RAW CRITERION SCORES
770 REM ***********************************************************
780 REM  READ IN PARAMETER VALUES
790 FOR I=FIRST TO LAST
800   IF I=1 THEN ZIP$=INPUT$(7, #1)
810   IF I=1 THEN CAND(I)=0
820   IF I>1 THEN INPUT #1, CAND(I)
830   FOR J=1 TO 41
840     INPUT #1, Y(J)
850   NEXT J
860 REM  COMPUTE X1
870   Z13=Y(24)-Y(25)
880   Z1222=(Y(20)*Y(21)*.001)+(Y(22)*Y(17))+(Y(23)*Y(18))
```

```
890   Z1221=(Y(11)*Y(12))/Y(19)
900   Z122=Z1221*Z1222*.001
910   Z1212=Y(14)+(Y(15)*Y(17))+(Y(16)*Y(18))
920   Z1211=(Y(11)*Y(12))/Y(13)
930   Z121=Z1211*Z1212*.001
940   Z12=Z121+Z122
950   Z1121B=(Y(3)/Y(4))^(1/Y(4))
960   Z1121F=(Y(5)/Y(6))^(1/Y(6))
970   Z1121M=(Y(7)/Y(8))^(1/Y(8))
980   Z1121C=(Y(9)/Y(10))^(1/Y(10))
990   Z1121=Z1121B*Z1121F*Z1121M*Z1121C
1000   Z112=1/(((Y(2)+2)/4)*Z1121)
1010   Z111=Y(1)
1020   Z11=Z111*Z112
1030   X1A(I)=1/(Z11+Z12+Z13)
1040   IF X1A(I)<XMIN(1) THEN XMIN(1)=X1A(I)
1050   IF X1A(I)>XMAX(1) THEN XMAX(1)=X1A(I)
1060 REM  COMPUTE X2
1070   Z23=Y(31)/Y(32)
1080   Z22=Y(29)*Y(30)
1090   Z21=((Y(26)^3)*Y(27))/Y(28)
1100   X2A(I)=1/(Z21*Z22*Z23)
1110   IF X2A(I)<XMIN(2) THEN XMIN(2)=X2A(I)
1120   IF X2A(I)>XMAX(2) THEN XMAX(2)=X2A(I)
1130 REM  COMPUTE X3
1140   Z33=0
1150   A=(Y(37)*Y(35))/Y(13)
1160   D=1
1170   FOR J=0 TO Y(37)-Y(38)
1180     B=A^J
1190     C=EXP(-A)
1200     IF J>1 THEN D=D*J
1210     Z33=Z33+((B*C)/D)
1220   NEXT J
1230   Z3222=1/((1/Y(13))+(1/Y(19)))
1240   Z32212=Y(22)+Y(23)
1250   Z32211=Y(15)+Y(16)
1260   Z3221=((Z32211/Y(13))+(Z32212/Y(19)))/((1/Y(13))+(1/Y(19)))
1270   Z322=(Y(35)*Z3221)/Z3222
1280   Z321=Y(35)+(Y(36)*30*24)
1290   Z32=(Z321-Z322)/Z321
1300   Z31=(Y(33)+(Y(34)/52))*Z112
1310   Z31=Z31+1
1320   X3A(I)=(Z32*Z33)/Z31
1330   IF X3A(I)<XMIN(3) THEN XMIN(3)=X3A(I)
1340   IF X3A(I)>XMAX(3) THEN XMAX(3)=X3A(I)
1350 REM  COMPUTE X4
1360   Z42=(Z3222*Y(39))/(Z3221*Y(40)*Y(41))
1370   Z41=Z33*Y(39)/Y(40)
1380   X4A(I)=Z41*Z42
1390   IF X4A(I)<XMIN(4) THEN XMIN(4)=X4A(I)
1400   IF X4A(I)>XMAX(4) THEN XMAX(4)=X4A(I)
```

```
1410   IF I MOD 200=0 THEN PRINT I
1420 NEXT I
1430 GOTO 460
```

Appendix C

SOURCE LISTING FOR BIG-X.BAS

```
10 REM ********************************************************
20 REM ********************************************************
30 REM
40 REM  BIG-X.BAS CONVERTS RAW CRITERION SCORES TO COMPARABLE
50 REM   UNITS, DENOTED AS BIG X'S.  IT OBTAINS THE RAW SCORES
60 REM    FROM B:LITTLEX1.DAT AND B:LITTLEX2.DAT.  RESULTS ARE
70 REM     STORED IN B:BIGX1.DAT AND B:BIGX2.DAT.
80 REM
90 REM ********************************************************
100 REM *******************************************************
110 REM  THIS MODULE PERFORMS INITIAL HOUSEKEEPING
120 REM *******************************************************
130 PRINT TIME$
140 OPTION BASE 1
150 FORM$="#.####"
160 DEFINT F,I,J,K
170 MAX=4753
180 DIM CAND(MAX)
190 DIM X1A(MAX)
200 DIM X2A(MAX)
210 DIM X3A(MAX)
220 DIM X4A(MAX)
230 DIM BIGX(15)
240 DIM XMIN(15)
250 DIM XMAX(15)
260 FOR I=1 TO 15
270   XMIN=9999
280   XMAX=0
290 NEXT I
300 REM *******************************************************
310 REM  THIS MODULE READS IN RAW CRITERION SCORES
320 REM *******************************************************
330 INPUT "PUT 1ST INPUT DISK IN B: AND PRESS ENTER";NIL$
340 OPEN "B:LITTLEX1.DAT" FOR INPUT AS #1
350 FOR I=1 TO 2377
360   INPUT #1, CAND(I),X1A(I),X2A(I),X3A(I),X4A(I)
370 NEXT I
380 CLOSE #1
390 INPUT "PUT 2ND INPUT DISK IN B: AND PRESS ENTER";NIL$
400 OPEN "B:LITTLEX2.DAT" FOR INPUT AS #1
410 FOR I=2378 TO MAX
```

```
420   INPUT #1, CAND(I),X1A(I),X2A(I),X3A(I),X4A(I)
430 NEXT I
440 CLOSE #1
450 REM *******************************************************
460 REM  THIS MODULE COMPUTES BIG X'S AND INTERSECTION X'S
470 REM *******************************************************
480 INPUT "PUT 1ST OUTPUT DISK IN B: AND PRESS ENTER";NIL$
490 OPEN "B:BIGX1.DAT" FOR OUTPUT AS #2
500 FOR I=1 TO MAX
510   IF I<>2378 THEN GOTO 550
520     CLOSE #2
530     INPUT "PUT 2ND OUTPUT DISK IN B: AND PRESS ENTER";NIL$
540     OPEN "B:BIGX2.DAT" FOR OUTPUT AS #2
550   PRINT #2, CAND(I);
560   X1=0
570   X2=0
580   X3=0
590   X4=0
600   X12=0
610   X13=0
620   X14=0
630   X23=0
640   X24=0
650   X34=0
660   X123=0
670   X124=0
680   X134=0
690   X234=0
700   X1234=0
710 REM  OBTAIN COUNTS FOR MARGINAL X'S
720   FOR J=1 TO MAX
730     IF X1A(I)>=X1A(J) THEN X1=X1+1
740     IF X2A(I)>=X2A(J) THEN X2=X2+1
750     IF X3A(I)>=X3A(J) THEN X3=X3+1
760     IF X4A(I)>=X4A(J) THEN X4=X4+1
770 REM  OBTAIN COUNTS FOR INTERSECTION X'S
780     IF X1A(I)>=X1A(J) AND X2A(I)>=X2A(J) THEN X12=X12+1
790     IF X1A(I)>=X1A(J) AND X3A(I)>=X3A(J) THEN X13=X13+1
800     IF X1A(I)>=X1A(J) AND X4A(I)>=X4A(J) THEN X14=X14+1
810     IF X2A(I)>=X2A(J) AND X3A(I)>=X3A(J) THEN X23=X23+1
820     IF X2A(I)>=X2A(J) AND X4A(I)>=X4A(J) THEN X24=X24+1
830     IF X3A(I)>=X3A(J) AND X4A(I)>=X4A(J) THEN X34=X34+1
840     IF X1A(I)>=X1A(J) AND X2A(I)>=X2A(J) AND X3A(I)>=X3A(J) THEN
X123=X123+1
850     IF X1A(I)>=X1A(J) AND X2A(I)>=X2A(J) AND X4A(I)>=X4A(J) THEN
X124=X124+1
860     IF X1A(I)>=X1A(J) AND X3A(I)>=X3A(J) AND X4A(I)>=X4A(J) THEN
X134=X134+1
870     IF X2A(I)>=X2A(J) AND X3A(I)>=X3A(J) AND X4A(I)>=X4A(J) THEN
X234=X234+1
880     IF X1A(I)>=X1A(J) AND X2A(I)>=X2A(J) AND X3A(I)>=X3A(J) AND
X4A(I)>=X4A(J) THEN X1234=X1234+1
```

```
890   NEXT J
900 REM  CONVERT COUNTS TO BIG X VALUES
910   BIGX(1)=X1/MAX
920   BIGX(2)=X2/MAX
930   BIGX(3)=X3/MAX
940   BIGX(4)=X4/MAX
950   BIGX(5)=X12/MAX
960   BIGX(6)=X13/MAX
970   BIGX(7)=X14/MAX
980   BIGX(8)=X23/MAX
990   BIGX(9)=X24/MAX
1000   BIGX(10)=X34/MAX
1010   BIGX(11)=X123/MAX
1020   BIGX(12)=X124/MAX
1030   BIGX(13)=X134/MAX
1040   BIGX(14)=X234/MAX
1050   BIGX(15)=X1234/MAX
1060   FOR J=1 TO 15
1070     IF BIGX(J)<XMIN(J) THEN XMIN(J)=BIGX(J)
1080     IF BIGX(J)>XMAX(J) THEN XMAX(J)=BIGX(J)
1090     PRINT #2, USING FORM$; BIGX(J);
1100   NEXT J
1110   PRINT #2, " "
1120   PRINT I;" ";TIME$
1130 NEXT I
1140 REM *********************************************************
1150 REM  THIS MODULE PERFORMS TERMINATION HOUSEKEEPING
1160 REM *********************************************************
1170 CLOSE #2
1180 LPRINT DATE$;" ";TIME$
1190 FOR I=1 TO 15
1200   LPRINT "XMIN(";I;")= ";XMIN(I);
1210   LPRINT " XMAX(";I;")= ";XMAX(I)
1220 NEXT I
1230 END
```

Appendix D

SOURCE LISTING FOR CF.BAS

```
10 REM *********************************************************
20 REM *********************************************************
30 REM
40 REM  CF.BAS COMPUTES CRITERION FUNCTION VALUES FOR
50 REM   CANDIDATE SYSTEMS BASED ON BIG X VALUES STORED
60 REM    IN TWO DATA FILES.  THE RESULTS ARE STOKED IN
70 REM     CF-OUT.DAT.
80 REM
90 REM *********************************************************
100 REM ********************************************************
110 REM  THIS MODULE PERFORMS INITIAL HOUSEKEEPING
120 REM ********************************************************
130 PRINT TIME$
140 OPTION BASE 1
150 FORM$="#.####"
160 DEFINT F,I,J,K
170 MAX=4753
180 DIM BIGX(15)
190 DIM A(15)
200 DIM BIGA(15)
210 CFMIN=9999
220 CFMAX=0
230 REM ********************************************************
240 REM  THIS MODULE COMPUTES RELATIVE WEIGHTS
250 REM ********************************************************
260 OPEN "BIGX1.DAT" FOR INPUT AS #1
270 OPEN "B:CF-OUT.DAT" FOR OUTPUT AS #2
280 FOR I=1 TO MAX
290   IF I<>2378 THEN GOTO 320
300     CLOSE #1
310     OPEN "BIGX2.DAT" FOR INPUT AS #1
320   INPUT #1, CAND
330   FOR J=1 TO 15
340     INPUT #1, BIGX(J)
350   NEXT J
360   PRINT #2, CAND;
370   BIGASUM=0
380   FOR J=1 TO 15
390 REM  COMPUTE A'S
400     IF BIGX(J)<=.01 THEN A(1)=1 ELSE A(1)=
ABS(LOG10(BIGX(J)))/2
```

```
410     IF BIGX(J)<=.5 THEN A(2)=.5 ELSE A(2)=BIGX(J)
420     A(3)=(2*BIGX(J))-(2*(BIGX(J) 2))
430     IF BIGX(J)<=.5 THEN A(4)=1-BIGX(J) ELSE A(4)=.5
440     A(5)=0
450     IF BIGX(J)<=.5 THEN A(6)=.3 ELSE A(6)=.6-((3/5)*BIGX(J))
460     A(7)=0
470     A(8)=.7
480     IF BIGX(J)<=.5 THEN A(9)=1-BIGX(J) ELSE A(9)=BIGX(J)
490     A(10)=0
500     FOR K=11 TO 15
510       A(K)=0
520     NEXT K
530 REM  NORMALIZE A'S VERTICALLY
540     ASUM=0
550     FOR K=1 TO 15
560       ASUM=ASUM+A(K)
570     NEXT K
580     BIGA(J)=A(J)/ASUM
590     BIGASUM=BIGASUM+BIGA(J)
600   NEXT J
610 REM  NORMALIZE A'S HORIZONTALLY
620   FOR J=1 TO 15
630     BIGA(J)=BIGA(J)/BIGASUM
640   NEXT J
650 REM  MODIFY THE A'S TO INCLUDE ALL HIGHER-ORDER INTERSECTIONS
660   BIGA(1)=BIGA(1)+BIGA(5)+BIGA(6)+BIGA(7)+BIGA(11)+BIGA(12)+
BIGA(13)+BIGA(15)
670   BIGA(2)=BIGA(2)+BIGA(5)+BIGA(8)+BIGA(9)+BIGA(11)+BIGA(12)+
BIGA(14)+BIGA(15)
680   BIGA(3)=BIGA(3)+BIGA(6)+BIGA(8)+BIGA(10)+BIGA(11)+BIGA(13)+
BIGA(14)+BIGA(15)
690   BIGA(4)=BIGA(4)+BIGA(7)+BIGA(9)+BIGA(10)+BIGA(12)+BIGA(13)+
BIGA(14)+BIGA(15)
700   BIGA(5)=BIGA(5)+BIGA(11)+BIGA(12)+BIGA(15)
710   BIGA(6)=BIGA(6)+BIGA(11)+BIGA(13)+BIGA(15)
720   BIGA(7)=BIGA(7)+BIGA(12)+BIGA(13)+BIGA(15)
730   BIGA(8)=BIGA(8)+BIGA(11)+BIGA(14)+BIGA(15)
740   BIGA(9)=BIGA(9)+BIGA(12)+BIGA(14)+BIGA(15)
750   BIGA(10)=BIGA(10)+BIGA(13)+BIGA(14)+BIGA(15)
760   BIGA(11)=BIGA(11)+BIGA(15)
770   BIGA(12)=BIGA(12)+BIGA(15)
780   BIGA(13)=BIGA(13)+BIGA(15)
790   BIGA(14)=BIGA(14)+BIGA(15)
800 REM ***********************************************************
810 REM  THIS ROUTINE COMPUTES CF
820 REM ***********************************************************
830   CF=0
840   FOR J=1 TO 4
850     CF=CF+(BIGA(J)*BIGX(J))
860   NEXT J
870   FOR J=5 TO 10
880     CF=CF-(BIGA(J)*BIGX(J))
```

```
890   NEXT J
900   FOR J=11 TO 14
910     CF=CF+(BIGA(J)*BIGX(J))
920   NEXT J
930   CF=CF-(BIGA(15)*BIGX(15))
940   PRINT #2, USING FORM$; CF
950   IF CF<CFMIN THEN WORST=CAND
960   IF CF<CFMIN THEN CFMIN=CF
970   IF CF>CFMAX THEN BEST=CAND
980   IF CF>CFMAX THEN CFMAX=CF
990   PRINT CAND;CF;WORST;CFMIN;BEST;CFMAX;TIME$
1000 NEXT I
1010 CLOSE #1
1020 CLOSE #2
1030 END
```

REFERENCES

Ackoff, R. L. "The Future of Operational Research is Past," Journal of the Operational Research Society, 30 (1979a), 93-104.

___________. "Resurrecting the Future of Operational Research," Journal of the Operational Research Society, 30 (1979b), 189-199.

Alavi, M. and Henderson, J. C. "An Evolutionary Strategy for Implementing a Decision Support System," Management Science, 27 (November 1981), 1309-1323.

Anscombe, F. J. and Aumann, R. J. "A Definition of Subjective Probability," Annals of Mathematical Statistics, 34 (March 1963), 199-205.

Asimow, M. Introduction to Design, Prentice-Hall, Inc., Englewood Cliffs, NJ, 1962.

Bard, Jonathan F. and Feinberg, Abe. "A Two-Phase Methodology for Technology Selection and System Design," IEEE Transactions on Engineering Management, 36 (February 1989), 28-36.

Barron, H. and Schmidt, C. P. "Sensitivity Analysis of Additive Multiattribute Value Models," Operations Research, 36 (January-February 1988), 122-127.

Bell, David E. "Mulitattribute Utility Functions: Decomposition Using Interpolation," Management Science, 25 (1979), 744-753.

___________, Keeney, R. H., and Raiffa, H. editors, Conflicting Objectives in Decisions, John Wiley, New York, 1977.

Besterfield, Dale H. Quality Control, Prentice-Hall, Englewood Cliffs, NJ, 1979.

Blanchard, Benjamin S. Logistics Engineering and Management, 3rd ed., Prentice-Hall, Englewood Cliffs, NJ, 1986.

Blin, Jean-Marie. "Fuzzy Sets in Multiple Criteria Decision-Making," in TIMS Studies in the Management Sciences, Starr and Zeleney (Eds.), 6 (1977), 129-146.

Boulding, Kenneth E. "General Systems Theory - The Skeleton of Science," Management Science, II (April 1956), 197-208.

Bowman, E. H. "Consistency and Optimality in Managerial Decision Making," Management Science, 9 (1963), 310-321.

Box, G. E. P. and Draper, N. Empirical Model Building and Response Surfaces, John Wiley, New York, 1986.

Bridgeman, P. W. Dimensional Analysis, Yale University Press, New Haven, CT, 1922.

Briskin, Lawrence E. "Establishing a Generalized Multiattribute Utility Function," in Multiple Criteria Decision Making, Cochrane and Zeleney (Eds.), University of South Carolina Press, Columbia, SC, 1973.

___________________. "A Method for Unifying Multiple Objective Functions," Management Science, 12 (June 1966), B-406 - B-416.

Brunswik, E. Systematic and Representative Design of Psychological Experiments, University of California Press, Berkeley, CA, 1947.

Buhl, H. R. Creative Engineering Design, Iowa State University Press, Ames, IA, 1961.

Carlsson, C. "Some Features of a Methodology for Dealing with Multi-Criteria Problems," in Theory and Practice of Multiple Criteria Decision Making, Carlsson and Kochetkov (Eds.), North-Holland, Amsterdam, 1983.

Cochrane, J. L. and Zeleney, M. (editors), Multiple Criteria Decision Making, University of South Carolina Press, Columbia, SC, 1973.

Charnes, A. and Cooper, W. W. Management Models and Industrial Applications of Linear Programming, Vols. I and II, John Wiley, New York, 1960.

Checkland, P. B. Systems Thinking, Systems Practice, John Wiley, New York, 1981.

Chehlaoui, Riad. "Capacity Planning Considerations in the Context of Design," Ph.D. Dissertation, University of Houston, 1975.

Cheng, Y. Y. M. and McInnis, B. "An Algorithm for Multiple Attribute, Multiple Alternative Decision Problems Based on Fuzzy Sets with Applications to Medical Diagnosis," IEEE Transactions on Systems, Man, and Cybernetics, SMC-10 (October 1980), 645-649.

Choo, E. U. and Wedley, W. C. "Optimal Criterion Weights in Multicriteria Decision Making," in Decision Making with Multiple Objectives, Haimes and Chankong (Eds.), Springer-Verlag, Berlin, 1985.

Choudhury, Aziz A. "Multiple Criterion Interaction Evaluation in

Design," Master's Thesis, University of Houston, 1979.

Churchman, C. W., Ackoff, R. L., and Arnoff, E. L. Introduction to Operations Research, John Wiley, New York, 1957.

Cleland, David I. and King, William R. Management: A Systems Approach, McGraw-Hill, New York, 1972.

Cohen, A. "Project Management: JSC's Heritage and Challenge," Space News Roundup, 28 (March 31, 1989), 3.

Cohen, Michael D.; Huber, George; Keeney, Ralph L.; Levis, Alexander H.; Lopes, Lola L.; Sage, Andrew P.; Sen, Subrata; Whinston, Andrew B.; Winkler, Robert L.; von Winterfeldt, Detlof; and Zadeh, Lofti. "Research Needs and the Phenomena of Decisionmaking and Operations," IEEE Transactions on Systems, Man, and Cybernetics, SMC-15(Nov/Dec 1985), 764-775.

Crawford, D. M., Huntzinger, B. C., and Kirkwood, C. W. "Multiobjective Decision Analysis for Transmission Conductor Selection," Management Science, 24 (December 1978), 1700-1709.

Dannenbring, D. G. and Starr, M. K. Management Science: An Introduction, McGraw-Hill, New York, 1981.

Dantzig, G. B. Linear Programming and Extensions, Princeton University Press, Princeton, NJ, 1963.

Dawes, R. M. "A Case Study of Graduate Admissions: Application of Three Principles of Human Decision Making," American Psychologist, 26 (1971), 180-188.

__________, and Corrigan, B. "Linear Models in Decision Making," Psychological Bulletin, 81 (February 1974), 95-106.

de Neufville, Richard, and Keeney, R. L. "Use of Decision Analysis in Airport Development for Mexico City" in Systems Planning and Design: Case Studies in Modeling, Optimization, and Evaluation, R. de Neufville and D. Marks (Eds.), Prentice-Hall, Englewood Cliffs, NJ, 1974.

__________________, and Marks, David, eds. Systems Planning and Design: Case Studies in Modeling, Optimization, and Evaluation, Prentice-Hall, Englewood Cliffs, NJ, 1974.

Department of Defense. Integrated Logistics Support Planning Guide, Government Printing Office, Washington, D.C., 1968.

Dewispelare, A. R. and Sage, A. P. "On Weakened Sufficiency Requirements for the Multiplicative Form of Value Function," IEEE Transactions on Systems, Man, and Cybernetics, SMC-9 (August 1979), 442-445.

Dixon, John R. Design Engineering: Inventiveness, Analysis and Decision Making, McGraw-Hill, New York, 1966.

Dyer, J. S. and Sarin, R. K. "Measurable Multiattribute Value Functions," Operations Research, 27 (July-August 1979), 810-822.

Easton, Allan. Complex Managerial Decisions Involving Multiple Objectives, Wiley, New York, 1973a.

____________. "One-of-a-Kind Decisions Involving Weighted Multiple Objectives and Disparate Alternatives," in Multiple Criteria Decision Making, Cochrane and Zeleney (Eds.), University of South Carolina Press, Columbia, SC, 1973b.

Ebert, R. J. and Kruse, T. E. "Bootstrapping the Security Analyst," Journal of Applied Psychology, 63 (1978), 110-119.

Eckenrode, R. T. "Weighting Multiple Criteria," Management Science, 12 (1965), 180-192.

Edwards, W. and Luce, R. D. "A Symposium on Utility Theory," presented at the Decision Sciences Institute Annual Meeting, Las Vegas, NV, November, 1988.

Eilon, S. "Structuring Unstructured Decisions," OMEGA, 13 (1985), 369-377.

Epstein, L. Ivan. "A Proposed Measure for Determining the Value of a Design," Operations Research, 5 (April 1957), 297-299.

Evans, J. R. "Sensitivity Analysis in Decision Theory," Decision Sciences, 15 (1984), 239-247.

Farquhar, P. H. "A Survey of Multiattribute Utility Theory and Applications" in Multiple Criteria Decision Making, Starr and Zeleney (Eds.), TIMS Studies in Management Sciences, Vol. 6, Amsterdam, North-Holland, 1977.

____________. "Advances in Multiattribute Utility Theory," Theory and Decision, 12 (1980), 381-394.

Feller, William. An Introduction to Probability Theory and Its Applications, Volume II, Wiley, New York, 1966.

Fishburn, P. C. Decision and Value Theory, Wiley, New York, 1964.

____________. "Additive Utilities with Finite Sets: Applications in the Management Science," Naval Research Logistics Quarterly, 14 (1967), 1-10.

____________. Utility Theory for Decision Making, Wiley, New York,

1970.

____________. "Interdependent Preferences on Finite Sets," Journal of Mathematical Psychology, 9 (1972a), 225-236.

____________. Mathematics of Decision Theory, Mouton, Paris, 1972b.

____________. "Foundations of Decision Analysis: Along the Way," Management Science, 35 (April 1989), 387-405.

Fletcher, R. and Powell, M. "A Rapidly Convergent Descent Method for Minimization," The Computer Journal, 6 (1963).

__________, and Reeves, C. "Function Minimization by Conjugate Gradients," The Computer Journal, 7 (1964).

Folkeson, John F. "Analysis of Interaction of Multiple Criteria with Application to Large Scale System Design," Ph.D. Dissertation, University of Houston, 1982.

Forrester, Jay W. Principles of Systems, The MIT Press, Cambridge, MA, 1968.

Gershon, Mark. "The Role of Weights and Scales in the Application of Multiobjective Decision Making," European Journal of Operational Research, 15 (February 1984), 244-250.

Goldberg, L. R. "Man Versus Model of Man: A Rationale, Plus Some Evidence for a Method of Improving on Clinical Inferences," Psychological Bulletin, 73 (1970), 422-432.

____________. "Man Versus Model of Man: Just How Conflicting in That Evidence?" Organizational Behavior and Human Performance, 16 (1976), 13-22.

Gosling, W. The Design of Engineering Systems, John Wiley, New York, 1962.

Green, P. E. and Srinivasan, V. "Conjoint Analysis in Consumer Research: Issues and Outlook," Journal of Consumer Research, 5 (September 1978), 103-123.

Greenwood, William T. Decision Theory and Information Systems: An Introduction to Management Decision Making, South-Western, Cincinnati, 1969.

Gregory, S. A. A Methodology for Systems Engineering, D. Van Nostrand, Princeton, NJ, 1966.

Hall, A. D. A Methodology for Systems Engineering, Van Nostrand Reinholt Co., New York, 1962.

__________. "Three-Dimensional Morphology of Systems Engineering," IEEE Transactions on Systems Science and Cybernetics SSC-5, No. 2 (Apr 1969), 156-160.

Hammond, K. R. "Probabilistic Functioning and the Clinical Method," Psychological Review, 62 (1955), 255-262.

Hamner, W. C. and Carter, P. L. "A Comparison of Alternative Production Management Coefficient Decision Rules," Decision Sciences, 6 (April 1975), 324-336.

Harrison, E. F. The Managerial Decision-Making Process, Houghton Mifflin, Boston, 1975.

Harvey, Charles M. "Value Functions for Infinite-Period Planning," Management Science, 32 (1986), 1123-1139.

______________. "Utility Functions for Infinite-Period Planning," Management Science, 34 (May 1988), 645-665.

Haupt, Ulrich. "Taking a New Perspective on Design," Astronautics and Aeronautics 16 (May 1978), 52-58.

Hill, Percy H. The Science of Engineering Design, Holt, Rinehart, and Winston, New York, 1970.

Hillier, F. S. and Lieberman, G. J. Introduction to Operations Research, 3rd edition, Holden-Day, Oakland, 1980.

Hoffman, Paul J. "The Paramorphic Representation of Clinical Judgement," Psychological Bulletin, 57 (1960), 116-131.

Hogarth, R. M. and Makridakis, S. "The Value of Decision Making in a Complex Environment: An Experimental Approach," Management Science, 27 (January 1981a), 93-107.

__________________________. "Forecasting and Planning: An Evaluation," Management Science, 27 (February 1981b), 115-133.

Holloway, Charles A. Decision Making Under Uncertainty: Models and Choices, Prentice-Hall, Englewood Cliffs, NJ, 1979.

Holzman, Albert G. "Engineering Design and Operations Research," Operations Research, 27 (March-April 1979), 420-422.

Hooke, R. and Jeeves, T. A. "'Direct Search' Solution of Numerical and Statistical Problems," Journal of the Association of Computing Machinery, (April 1961).

Howard, Ronald A. "An Assessment of Decision Analysis," Operations Research, 28 (January-February 1980), 4-27.

_______________. "Decision Analysis: Practice and Promise," Management Science, 34 (June 1988), 679-695.

Huber, G. P. "Multi-Attribute Utility Models: A Review of Field and Field-Like Studies," Management Science, 20 (June 1974a), 1393-1401.

___________. "Methods for Quantifying Subjective Probabilities and Multi-Attribute Utilities," Decision Sciences, 5 (July 1974b), 430-458.

Hurst, E. G. and McNamara, A. B. "Heuristic Scheduling in a Woolen Mill," Management Science, 14 (December 1967), B-182-203.

Ignizio, J. P. Goal Programming and Extensions, Lexington Books, Lexington, MA, 1976.

Janis, Irving L. and Mann, Leon. Decision Making, MacMillan, The Free Press, New York, 1977.

Johnson, E. J. and Payne, J. W. "Effort and Accuracy in Choice," Management Science, 31 (April 1985), 395-414.

Johnson, R. A., Kast, F. E., and Rosenzweig, J. E. "Systems Theory and Management," Management Science, 10 (January 1964), 367-384.

____. The Theory and Management of Systems, 2nd ed., McGraw-Hill, New York, 1967.

Jones, J. C. Design Methods: Seeds of Human Futures, John Wiley, New York, 1970.

Kast, F. E. and Rosenzweig, J. E. Organization and Management: A Systems Approach, McGraw-Hill, New York, 1970.

Keen, Peter G. W. "The Evolving Concept of Optimality," in TIMS Studies in the Management Sciences, Starr and Zeleney (Eds.), 6 (1977), 31-57.

_______________. "Adaptive Design for D.S.S.," Database, 12 (Fall 1980), 15-25.

Keeney, Ralph L. "An Illustrated Procedure for Assessing Multiattributed Utility Function," Sloan Management Review, 14 (1972), 37-50.

_______________. "Concepts of Independence in Multiattribute Utility Theory," in Multiple Criteria Decision Making, Cochrane and Zeleney (Eds.), University of South Carolina Press, Columbia, SC, 1973a.

_______________. "A Decision Analysis with Multiple Objectives: The Mexico City Airport," Bell Journal of Economics and Management Science, 4 (Spring 1973b) 101-117.

______________. "A Group Preference Axiomatization with Cardinal Utility," Management Science, 23 (1976a), 140-145.

______________, and Raiffa, H. Decisions with Multiple Objectives: Preferences and Value Tradeoffs, Wiley, New York, 1976b.

______________. "A Survey of Multiattribute Utility Theory," in IE Conference Proceedings, (Fall 1978), 147-153.

______________. "Decision Analysis: An Overview," Operations Research, 30 (September-October 1982), 803-838.

______________. "Structuring Objectives for Problems of Public Interest," Operations Research, 36 (May-June 1988), 396-405.

Kirkwood, C. W. "Pareto Optimality and Equity in Social Decision Analysis," IEEE Transactions on Systems, Man, and Cybernetics, 9 (1979), 89-91.

______________, and Sarin, R. K. "Preference Conditions for Multiattribute Value Functions," Operations Research, 28 (1980), 225-232.

Kleinbaum, D. G., Kupper, L. L., and Muller, K. E. Applied Regression Analysis and Other Multivariable Methods, 2nd edition, PWS-Kent, Boston, 1988.

Kline, M. B. "Systems Engineering Methodology: A Conceptual Phase Decision Model," in A General Survey of Systems Methodology, L. Troncale (Ed.), Louisville, KY; The Society for General Systems Research, Vol. 2, 608-617.

Kunreuther, H. "Extensions of Bowman's Theory on Managerial Decision-Making," Management Science, 15 (1969), B-415-439.

La Valle, Irving H. Fundamentals of Decision Analysis, Holt, Rinehart, and Winston, 1978.

Lee, Sang M. Goal Programming for Decision Analysis, Auerbach, Philadelphia, 1972.

Leon, A. "A Comparison of Eight Known Optimizing Procedures," in Recent Advances in Optimization Techniques, Lavi and Vogl (Eds.), John Wiley, New York, 1966.

Lifson, Melvin K. Decision and Risk Analysis for Practicing Engineers, Cahners Books, Boston, 1972.

Loeve, M. Probability Theory, Van Nostrand, New York, 1963.

MacCrimmon, Kenneth R. "An Overview of Multiple Objective Decision

Making," in Multiple Criteria Decision Making, Cochrane and Zeleney (Eds.), University of South Carolina Press, Columbia, SC, 1973.

Makridakis, S., Andersen, A., Carbone, R., Fildes, R., Hibon, M., Lewandowski, R., Newton, J., Parzen, R., and Winkler, R. "The Accuracy of Extrapolation (Time Series) Methods: Results of a Forecasting Competition," Journal of Forecasting, 1 (1982), 111-153.

McCumber, W. M. "An Interactive System Design Heuristic," Ph.D. Dissertation, University of Houston, 1987.

Mehrez, A. and Sinuany-Stern, Z. "Resource Allocation to Interrelated Risky Projects Using a Multiattribute Utility Function," Management Science, 29 (1983), 430-439.

Meredith, Dale D. et al. Design and Planning of Engineering Systems, Prentice-Hall, Englewood Cliffs, NJ, 1973.

Middendorf, W. H. Engineering Design, Allyn and Bacon, Boston, MA, 1969.

Miller, James R., III. Professional Decision-Making--a Procedure for Evaluating Complex Alternatives, Praeger Publishers, New York, 1970.

Nadler, Gerald. "An Investigation of Design Methodology," Management Science, 13 (June 1967), B-642 - B-655.

______________. "Systems Methodology and Design," IEEE Transactions on Systems, Man, and Cybernetics, SMC-15 (November/December 1985), 685-697.

National Aeronautics and Space Administration (NASA), Space Shuttle News Reference, 1989.

Nutt, Paul C. "Comparing Methods for Weighting Decision Criteria," Omega, 8 (1980), 163-172.

Olenik, S. C. and Haimes, Y. Y. "A Hierarchical Multiobjective Framework for Water Resources Planning," IEEE Transactions on Systems, Man, and Cybernetics, SMC-9 (September 1979), 534-544.

Ostrofsky, Benjamin. "Theory for Criterion Function Synthesis in Design," Unpublished Ph.D. Dissertation, University of California, Los Angeles, 1968.

__________________. "Morphology of Design of Aerospace Systems with Inclusion of Human Resource Factors," AFOSR Grant # 77-3148, 1977a.

__________________. "An Annotated Bibliography on Morphology of Design with Inclusion of Human Factors," University of Houston, University Park, March, 1977b.

__________________. "Application of a Structured Design Process For Proper Inclusion of Human Resources in the Design of a Power Unit Support Stand," University of Houston, University Park, 1977c.

__________________. Design, Planning, and Development Methodology, Prentice-Hall, Inc., Englewood Cliffs, NJ, 1977d.

__________________, Hakim, Ramzi S., Folkeson, John R., Jr., "An Application of a Design Morphology to Aerospace Systems with the Inclusion of Human Factors," University of Houston, 1978.

__________________. "Preliminary Activities in the Development of MX Maintenance Control Through Use of a Design Morphology," University of Houston, University Park, 1979.

__________________, Donaghey, Charles E., Marquina, Nelson E., Kiessling, E. A., "Application of a Design Morphology to the MX/OCC Definition of a Fault Detection and Dispatch System," University of Houston, University Park, September, 1980.

__________________, Law, Japhet, Kiessling, Ernest A., Putcha, Mallik, "MX Maintenance Management System Definition," University of Houston, University Park, September, 1981.

__________________, Kiessling, E. A., "A Review and Analysis of IBM Business System Planning (BSP) and an Information Resource Management (IRM) Application," University of Houston, 1984.

__________________, Kiessling, E. A., Hixon, S. D., Wallace, K., "Productivity Investigation of Accounting and Finance (ACF) Activities at the Space Division, Air Force Systems Command," University of Houston, January, 1986.

__________________. "Models for Criterion Function Synthesis," MSAS 8365 Class Notes, University of Houston, Spring 1987.

__________________. Personal communication, 1988.

Parzen, Emanuel. Modern Probability Theory and Its Applications, Wiley, New York, 1960.

Peschke, R. E. "A Structured Optimization Method for Information Resource Management," Ph.D. Dissertation, University of Houston, 1986.

Rabins, M., Ardayfio, D., Fenves, S., Seireg, A., Nadler, G., Richardson, H., and Clark, H. "Design Theory and Methodology - A New Discipline," Mechanical Engineering, (August 1986), 23-27.

Raiffa, Howard. Decision Analysis, Addison-Wesley, Reading, MA, 1968.

Rao, C. R. Linear Statistical Inference and Its Applications, John Wiley, New York, 1965.

Ross, Sheldon. A First Course in Probability, 3rd edition, Macmillan, New York, 1988.

Roy, Bernard. "A Conceptual Framework for a Prescriptive Theory of 'Decision-Aid'," in TIMS Studies in the Management Sciences, Starr and Zeleney (Eds.), 6 (1977), 179-210.

Rubinstein, Moshe F. Patterns of Problem Solving, Prentice Hall, Englewood Cliffs, NJ, 1975.

__________________. Concepts of Problem Solving, Prentice Hall, Englewood Cliffs, NH, 1980.

__________________. Tools for Thinking and Problem Solving, Prentice-Hall, Englewood Cliffs, NJ, 1986.

Saaty, T. L. "A Scaling Method for Priorities in Hierarchical Structures," Journal of Mathematical Psychology, 15 (1977), 234-281.

__________. The Analytic Hierarchy Process, McGraw-Hill, New York, 1980.

__________. Decision Making for Leaders, Van Nostrand Reinhold, Belmont, CA, 1982.

__________. "The Analytic Hierarchy Process," presented at the University of Houston, Houston, TX, 1988.

Sage, Andrew P. Methodology for Large-Scale Systems, McGraw-Hill, New York, 1977.

_____________. "Behavioral and Organizational Considerations in the Design of Information Systems and Processes for Planning and Decision Support," IEEE Transactions on Systems, Man, and Cybernetics SMC-11 (September 1981), 640.

_____________, editor. System Design for Human Interaction, IEEE Press, 1986.

Sakawa, M. and Seo, F. "Interactive Multiobjective Decision Making for Large-Scale Systems and Its Application to Environmental Systems," IEEE Transactions on Systems, Man, and Cybernetics, SMC-10 (December 1980), 796-806.

Savage, L. J. The Foundations of Statistics, John Wiley, New York, 1954.

Seger, James K. "Operational Availability of Intermittent Missions," Logistics Spectrum, 23 (Spring 1989), 29-31.

Shestakov, O. A. "SIMS, An Interactive Multi-Criteria Search System,"

in Theory and Practice of Multiple Criteria Decision Making, Carlsson and Kochetkov (Eds.), North-Holland, Amsterdam, 1983.

Shull, F. A., Jr., Delbecq, A. L. and Cummings, L. L. Organizational Decision Making, McGraw-Hill, New York, 1970.

Simon, Harold A. A Student's Introduction to Engineering Design, Pergamon Press, New York, 1975.

Simon, Herbert A. "A Behavioral Model of Rational Choice," Quarterly Journal of Economics, (February 1955), 99-118.

_______________, and Newell, A. "Heuristic Problem Solving: the Next Advance in Operations Research," Operations Research, (January-February 1958), 1-10.

_______________. The New Science of Management Decision, Revised ed., Prentice-Hall, Englewood Cliffs, NJ, 1977.

_______________, et al. "Decision Making and Problem Solving," Interfaces, 17 (September-October 1978), 11-31.

Simons, J. V., Jr. and Ostrofsky, B. "Research in Multiple Criteria Design Optimization," Proceedings of the 23rd International Logistics Symposium, Society of Logistics Engineers, Huntsville, AL, August 1988.

Smith, Gerald F. "Towards a Heuristic Theory of Problem Structuring," Management Science, 34 (December 1988), 1489-1506.

Starr, Martin K. "Design Morphologies," DMG-DRS Journal: Design Research and Methods, 7 (April-June 1973), 98.

_______________, and Zeleney, Milan. "MCDM - State and Future of the Arts," TIMS Studies in the Management Sciences 6 (1977), 111-127.

_______________, and Greenwood, Leonard H. "Normative Generation of Alternatives with Multiple Criteria Evaluation," in TIMS Studies in the Management Sciences, 6 (1977), 111-127.

Taubert, W. H. "A Search Decision Rule for the Aggregate Scheduling Problem," Management Science, 14 (1968), 343-359.

Taylor, Robert L. "Consistent Multiattributed Decision Procedures," abridgement in Multiple Criteria Decision Making, Cochrane and Zeleney (Eds.), University of South Carolina Press, Columbia, SC, 1973.

Todd, F. J. "A Methodological Study of Clinical Judgement," unpublished Ph.D. dissertation, University of Colorado, 1954.

Todd, Peter, and Benbasat, Izak, "Process Tracing Methods in Decision Support Systems Research: Exploring the Black Box," MIS Quarterly,

(December 1987).

Tompkins, J. A. "Industrial Engineering and Manufacturing Design: IEs Urged to Step Forward and Claim Design Turf," Industrial Engineering, 21 (June 1989), 44-45.

Tribus, Myron. Rational Descriptions, Decisions and Designs, Pergamon Press, New York, 1969.

Tversky, A. and Kahneman, D. "Judgement Under Uncertainty: Heuristics and Biases," Science, 185 (September 27, 1974), 1124-1131.

U. S. Air Force. "Systems Engineering Management Procedures," U.S. Air Force Systems Command Manual (AFSCM) 375-5, Government Printing Office, Washington, D.C., 1966.

______________. "Program Management," U. S. Air Force Systems Command Regulation (AFSCR) 800-2, Government Printing Office, Washington, D.C., 18 October 1974.

von Neumann, John, and Morgenstern, Oskar. The Theory of Games and Economic Behavior, Princeton University Press, Princeton, NJ, 1944.

Van Gigch, John P. Applied General Systems Theory, 2nd edition, Harper and Row, New York, 1978.

Vidosic, Joseph. Elements of Engineering Design, Ronald Press, New York, 1969.

Wagner, H. M. Principles of Operations Research, Prentice-Hall, Englewood Cliffs, NJ, 1975.

Wallace, Ken M. An Introduction to the Design Process, Department of Engineering, University of Chicago, October 1978.

______________. Personal communication, 1988.

White, C. C. and Sage, A. P. "A Multiple Objective Optimization Approach to Choice Making," IEEE Transactions on Systems, Man, and Cybernetics, SMC-10 (June 1980), 315-326.

Wilks, Samuel S. Mathematical Statistics, John Wiley, New York, 1962.

Wilson, Warren E. Concepts of Engineering System Design, McGraw-Hill, New York, 1965.

Woodson, T. T., Introduction to Engineering Design, McGraw-Hill, New York, 1966.

Wu, Anthony B. "Strategic Design and Decision Support Methodology for Large Scale Advanced Technological Systems," Ph.D. Dissertation, University of Houston, 1983.

Wymore, Wayne A. A Mathematical Theory of Systems Engineering - the Elements, Wiley, New York, 1967.

______________. Systems Engineering Methodology for Interdisciplinary Teams, Wiley, New York, 1976.

Yager, Ronald R. "A New Methodology for Ordinal Multiobjective Decisions Based on Fuzzy Sets," Decision Sciences, 12 (1981), 589-600.

Zeleney, M. "A Selected Bibliography of Works Related to Multiple Criteria Decision Making," in Multiple Criteria Decision Making, Cochrane and Zeleney (Eds.), University of South Carolina Press, Columbia, SC, 1973.

__________. "A Concept of Compromise Solutions and Method of the Displaced Ideal," Computers and Operations Research, 1 (1974), 479-496.

__________. "The Theory of the Displaced Ideal," Multiple Criteria Decision Making, Springer-Verlag, 1976a, 153.

__________. "The Attribute-Dynamic Attitude Model (ADAM)," Management Science 23 (1976b), 12-26.

__________. "Adaptive Displacement of Preferences in Decision Making," TIMS Studies in the Management Sciences 6 (1977), 146-158.

__________. Computers and Operations Research: Special Issue on Mathematical Programming with Multiple Objectives, 7,1980.

__________. "Descriptive Decision Making and Its Applications," Applications of Management Science 1 (1981a), 327-388.

__________. "A Case Study in Multiobjective Design: De Novo Programming," Criteria Analysis: Operational Methods, ed by Nijkamp and Sprok, Grover Press, London, 1981b.

__________. Multiple Criteria Decision Making, McGraw-Hill, New York, 1982.

__________. "Multicriterion Design of High-Productivity Systems," in Multiple Criteria Decision Making, Zeleney (Ed.), Jay Press, Greenwich, CN, 1984.

Zumwalt, Crane. "Methodology for Optimal Selection of Off-Shore Drilling Vessels," Ph.D. Dissertation, University of Houston, 1979.